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Problem 2.1: Temperature scales

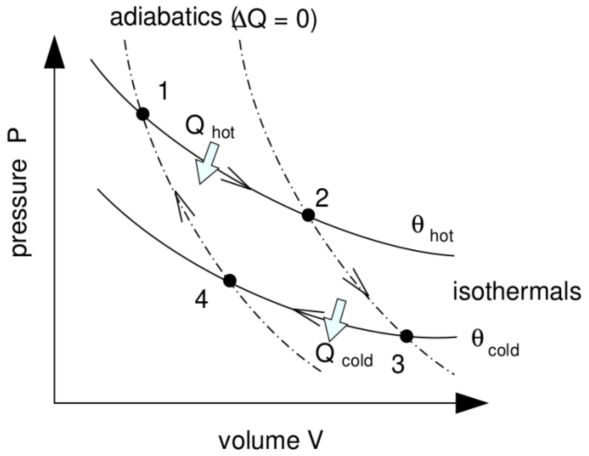
[**Oral** | 4 pt(s)]

ID: ex_temperature_scales:sm2627

Learning objective

In the lecture we have seen two temperature scales: In the ideal gas temperature scale we find the temperature via $P\frac{V}{N} =: k_B\Theta$, while in the thermodynamic temperature scale we can measure a temperature against a reference via a Carnot machine using the relation $\frac{T_2}{T_1} = \frac{Q_2}{Q_1}$. In this exercise you show the equality of these two temperature scales.

Show the equivalence of the ideal gas temperature scale Θ, and the thermodynamic scale T, by performing a Carnot cycle on an ideal gas. The Carnot cycle is sketched on the right. The ideal gas satisfies $PV = Nk_B\Theta$, and its internal energy U is a function of Θ only. The thermodynamic scale is defined by $\frac{T_2}{T_1} = \frac{Q_2}{Q_1}$.



a) Calculate the heat flowing into Q_{hot} and flowing out of Q_{cold} the Carnot machine as a function of Θ_{hot} and Θ_{cold} and the corresponding volumes. **2**pt(s)

b) Consider the adiabatic expansion and compression and show that $V_3/V_2 = V_4/V_1$. **1**pt(s)

c) Show that $Q_{\text{hot}}/Q_{\text{cold}} = \Theta_{\text{hot}}/\Theta_{\text{cold}}$ **1**pt(s)

Problem 2.2: Adiabatic process

[**Oral** | 3 pt(s)]

ID: ex_adiabatic_process:sm2627

Learning objective

In this exercise we consider general adiabatic processes. We use them as an example to study how one can derive general relation between thermodynamic response functions.

We consider a macroscopic system of temperature T and volume V. An adiabatic process exchanges no heat with the environment $\delta Q = 0$. We consider reversible adiabatic processes, these therefore leave the entropy S constant.

a) Show that 1pt(s)

$$\left(\frac{\partial V}{\partial T}\right)_S = -\frac{C_V/T}{\left(\frac{\partial P}{\partial T}\right)_V}\,, \tag{1}$$

with the isochoric heat capacity $C_V = \left(\frac{\delta Q}{dT}\right)_V$.

b) Show that 1pt(s)

$$\left(\frac{\partial P}{\partial T}\right)_S = \frac{C_P}{V\alpha T}\,, \tag{2}$$

with the thermal expansion coefficient $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$ and the isobaric heat capacity $C_P = \left(\frac{\delta Q}{dT}\right)_P$.

c) Use the previous two results to show that the adiabatic compressibility fulfills 1pt(s)

$$\kappa_S := -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_S = \frac{C_V}{C_P}\frac{\alpha}{\left(\frac{\partial P}{\partial T}\right)_V} \tag{3}$$

Problem 2.3: Van der Waals fluid

[**Oral** | 4 pt(s)]

ID: ex_Van_der_Waals_fluid:sm2627

Learning objective

In this exercise you consider a van der Waals fluid, that is a fluid which has the van der Waals equation as its equation of state. Compared to the ideal gas the more complicated state equation makes calculations more difficult but still analytically tractable. This additional complexity also allows for phenomena not described by the ideal gas, such as instability related to the liquid-vapour phase transition.

The empirical equation of state for a van der Waals fluid is given by

$$\left(P + \frac{N^2 a}{V^2}\right)(V - Nb) = Nk_B T\,, \tag{4}$$

in which the constants a and b are constants with proper units.

a) Use the equation of state to determine the *free energy density* $f(T, v) = F(T, V, N)/N$ with volume per particle $v = V/N$ up to a factor $f_0(T)$. Find the value of this factor by matching it to the free energy density of the ideal gas 1pt(s)

$$f_{\text{ideal}}(T, v) = -k_B T \ln(\Lambda^{-3} v)\,, \quad \text{with} \quad \Lambda = \sqrt{\frac{2\pi\hbar^2}{mk_B T}}\,, \tag{5}$$

in the limit of large v.

b) Calculate the heat capacities C_V and C_P and the compressibilities κ_T and κ_S. 2pt(s)

Hint: Calculate the entropy and combine its derivatives and the derivatives of the equation of state.

c) Compute the spinodal for the van der Waals fluid, that is compute the curve $P(v)$ where the fluid becomes mechanically unstable ($(\partial P/\partial v)_T = 0$). Find the critical point (in addition $(\partial^2 P/\partial v^2)_T = 0$) and give (T_c, P_c, v_c). 1pt(s)