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Problem 3.1: Carnot Cycle with Ideal Gas

[**Written** | 2 pt(s)]

ID: ex_Carnot_cycle_ideal_gas:sm2324

Learning objective

In this problem you will compute the efficiency of the Carnot cycle which is the upper limit on the efficiency of any classical thermodynamic engine which transforms heat into work. The cycle is reversible, the inverse process is the most efficient heat pump.

Show that the efficiency of a Carnot machine, when an ideal gas is used as a medium, is given by the following:

$$\eta = 1 - \frac{T_2}{T_1}. \tag{1}$$

Calculate the work and heat transfer for each step of the cycle

Problem 3.2: Otto Cycle

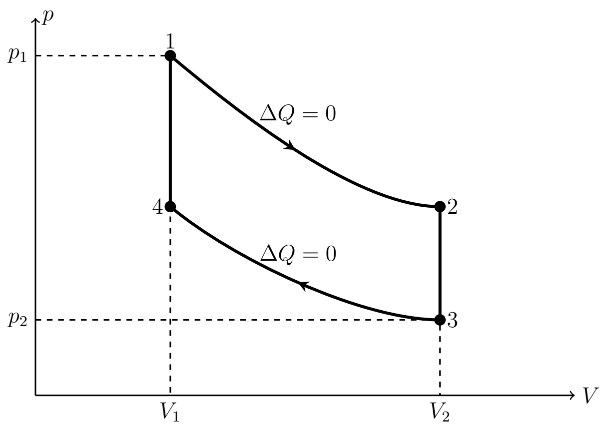
[**Oral** | 2 pt(s)]

ID: ex_Otto_cycle:sm2324

Learning objective

In the following problem we study a thermodynamic cycle which describes a simplified 4-stroke engine powered throught a gas-air (fuel-air) mixture, commonly used in automobiles.

Convert the $p - V$ diagram shown bellow into a $T - S$ diagram.



Calculate the efficiency of this machine for an ideal gas as medium and for c_V constant.
Show that $\eta_{Otto} < \eta_{Carnot}$.

Hints:

- The difference in entropy:

$$S - S_0 = \int_{T_0}^{T} c_V \frac{dT}{T} + \int_{V_0}^{V} R \frac{dV}{V}. \tag{2}$$

- For an adiabatic process:

$$TV^{R/c_V} = T_0 V_0^{R/c_V}. \tag{3}$$

Problem 3.3: Equation of State for magnetic Substances [**Written** | 2 pt(s)]

ID: ex_equation_of_state_for_magnetic_substances:sm2324

Learning objective

Let us revisit this excercise as it is useful for the next. Here you will establishe a connection between magnetization and equation of state for internal energy in the presence of uniform magnetic field. Further, using the Curie-law, you will show that the internal energy is independent of magnetization, and only depends on temperature.

Consider a homogeneous magnetic field $\mathbf{H}$ is created by a long coil. Then, an isotropic, magnetic material shall be placed in the center of the coil. The reversible work performed by the coil onto the material is given by

$$\delta A = H\, dM\,,$$

at unity volume. Here M is the magnetization of the material. Since we are dealing with an isotropic material, the vector nature of H and M can be neglected.

a) Write down the entropy of the system as $S = S(T, H)$ and with this derive the relationship between the magnetization $M = M(T, H)$ (thermic equation of state) and the internal energy $U = U(T, H)$ (caloric equation of state). **1** pt(s)

Hint: Using $S = S(T, H)$ derive the exact differential form dS and compare it with $dS = \delta Q/T$. This comparison provides you a relationship between entropy, internal energy and magnetization as follows:

$$\left.\frac{\partial S}{\partial T}\right|_H = \left[\frac{1}{T}\left.\frac{\partial U}{\partial T}\right|_H - \frac{H}{T}\left.\frac{\partial M}{\partial T}\right|_H\right], \qquad \left.\frac{\partial S}{\partial H}\right|_T = \left[\frac{1}{T}\left.\frac{\partial U}{\partial H}\right|_T - \frac{H}{T}\left.\frac{\partial M}{\partial H}\right|_T\right].$$

Now calculate the second derivative of S which holds the relation $\frac{\partial^2 S}{\partial H \partial T} = \frac{\partial^2 S}{\partial T \partial H}$, and obtain the result

$$\left.\frac{\partial U}{\partial H}\right|_T = T\left.\frac{\partial M}{\partial T}\right|_H + H\left.\frac{\partial M}{\partial H}\right|_T.$$

This is the relation between caloric and thermic equation of states.

b) A paramagnetic substance fulfills the Curie-law **1** pt(s)

$$M = K \cdot \frac{H}{T}\,,$$

with K being a material dependent constant. Show that U only depends on T.

Problem 3.4: Magnetic Carnot Machine [**Oral** | 3 pt(s)]

ID: ex_magnetic_Carnot_machine:sm2324

Learning objective

In this problem, we study a Carnot cycle with an isotropic paramagnetic material as a medium. We show that the efficiency is quivalent to that of a Carnot cycle with an ideal gas.

Consider a paramagnetic material as in Problem 3.3, which has a constant heat capacity $c_M = (\partial U/\partial T)_M$. The material shall be used as a Carnot machine between two heat reservoirs of temeratures $T_2 > T_1$.

a) Find the equation of the adiabates of the system and sketch a cycle for the machine in a $T - M$ diagram. In which direction should the cycle run, in order to perform work? **1**pt(s)

b) Calculate the work performed by the machine in one cycle. **1**pt(s)

c) Compute the efficiency of the machine. **1**pt(s)