

Problem 2.1: Internal Energy

[Written | 2 pt(s)]

ID: ex_internal_energy:sm2324

Learning objective

In this exercise, you will numerically calculate the internal energies for thermal equilibrium of two systems which are in diathermic contact.

Given there are two systems Σ_1 and Σ_2 , see figure 1 with the equations of state

$$\Sigma_1 : U_1 = \frac{3}{2}n_1RT_1 \qquad \Sigma_2 : U_2 = \frac{5}{2}n_2RT_2 ,$$

which are in diathermic contact. Here n is the number of moles, R is the gas constant with $R = 8.314\text{J}/(\text{K} \cdot \text{mol})$.

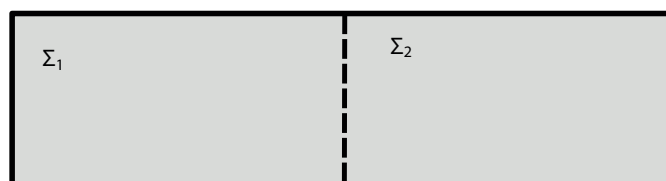


Abbildung 1: Two systems in diathermic contact.

- a) The total energy of the combined system shall be U . Determine the internal energies U_1 and U_2 for Σ_1 and Σ_2 , respectively, for thermal equilibrium. As an example, insert the values $n_1 = 2$ mol, $n_2 = 3$ mol and $U = 24000$ J. 1pt(s)
- b) Now, instead of the total energy, the initial temperatures T_1 and T_2 of Σ_1 and Σ_2 , respectively, are given. Determine U_1 , U_2 and T for thermal equilibrium, with $T_1 = 250$ K, $T_2 = 300$ K, $n_1 = 2$ mol and $n_2 = 3$ mol. 1pt(s)

Problem 2.2: Ideal Gas, Part 1

[Written | 1 pt(s)]

ID: ex_ideal_gas_part1:sm2324

Learning objective

Ideal gas law helps us to understand how changes in pressure, volume, temperature, and the number of moles affect the behavior of gases. In this exercise, you will derive such a relation and find out that the internal energy of the system is independent of volume and depends only on temperature for ideal gas.

From the lecture we know the following relations

$$\begin{aligned}dU &= \delta Q - \delta W, \\ \delta Q &= C_V dT, \\ \delta W &= p dV.\end{aligned}$$

Hence

$$\left. \frac{\partial U}{\partial V} \right|_T = -p.$$

This is obviously wrong for the ideal gas, where $U = U(T)$, i.e. U is independent of V . Where is the mistake?

Problem 2.3: Ideal Gas, Part 2

[Written | 5 pt(s)]

ID: ex_ideal_gas_part2:sm2324

Learning objective

In this exercise, you will explore various aspects of ideal gases and their behavior in different thermodynamic processes such as isobaric, isothermic, and isochoric. Additionally, you will learn about the reversible and irreversible processes, and examine adiabatic expansions. At last, you will calculate the expression for the entropy change during a free expansion of a gas.

- The ideal gas law is $PV = nRT$, where n again is the number of moles of gas and R the gas constant. A process equation is a simple function of, usually, two variables of P , V , T , which remains constant throughout the process. Find isobaric, isothermic and isochoric process equations. 1pt(s)
- Apply the ideal gas law to get the adiabatic process equation $PV^\gamma = const.$, where $\gamma = C_P/C_V$. 1pt(s)
- What is the meaning of a reversible process? Explain how to perform an adiabatic process in an reversible procedure. 1pt(s)
- Consider the free expansion of a gas, figure 2. This is also an adiabatic expansion, but of irreversible nature. Why? 1pt(s)
- During a free expansion, δQ , dU and δW are zero, respectively, but still the entropy increases. Find the expression for the increasing entropy. 1pt(s)

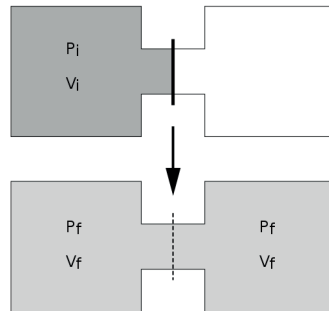


Abbildung 2: Upper part: Two chambers, separated by a valve with initial pressure P_i and initial volume V_i . Lower part: System after opening the valve with final pressure P_f and final volume V_f .

Problem 2.4: Equation of State for magnetic Substances

[Written | 2 pt(s)]

ID: ex_equation_of_state_for_magnetic_substances:sm2324

Learning objective

In this exercise, you will establish a connection between magnetization and equation of state for internal energy in the presence of uniform magnetic field. Further, using the Curie-law, you will show that the internal energy is independent of magnetization, and only depends on temperature.

Consider a homogeneous magnetic field \mathbf{H} is created by a long coil. Then, an isotropic, magnetic material shall be placed in the center of the coil. The reversible work performed by the coil onto the material is given by

$$\delta A = H dM,$$

at unity volume. Here M is the magnetization of the material. Since we are dealing with an isotropic material, the vector nature of H and M can be neglected.

- a) Write down the entropy of the system as $S = S(T, H)$ and with this derive the relationship between the magnetization $M = M(T, H)$ (thermic equation of state) and the internal energy $U = U(T, H)$ (caloric equation of state). 1pt(s)

Hint: Using $S = S(T, H)$ derive the exact differential form dS and compare it with $dS = \delta Q/T$. This comparison provides you a relationship between entropy, internal energy and magnetization as follows:

$$\left. \frac{\partial S}{\partial T} \right|_H = \left[\frac{1}{T} \left. \frac{\partial U}{\partial T} \right|_H - \frac{H}{T} \left. \frac{\partial M}{\partial T} \right|_H \right], \quad \left. \frac{\partial S}{\partial H} \right|_T = \left[\frac{1}{T} \left. \frac{\partial U}{\partial H} \right|_T - \frac{H}{T} \left. \frac{\partial M}{\partial H} \right|_T \right].$$

Now calculate the second derivative of S which holds the relation $\frac{\partial^2 S}{\partial H \partial T} = \frac{\partial^2 S}{\partial T \partial H}$, and obtain the result

$$\left. \frac{\partial U}{\partial H} \right|_T = T \left. \frac{\partial M}{\partial T} \right|_H + H \left. \frac{\partial M}{\partial H} \right|_T.$$

This is the relation between caloric and thermic equation of states.

- b) A paramagnetic substance fulfills the Curie-law 1pt(s)

$$M = K \cdot \frac{H}{T},$$

with K being a material dependent constant. Show that U only depends on T .