Prof. Dr. Hans-Peter Büchler
Institute for Theoretical Physics III, University of Stuttgart

January 10th, 2023
WS 2023/24

Problem 11.1: Ideal Paramagnet (Part 1) [**Written** | 4 pt(s)]

ID: ex_ideal_paramagnet_part_1_sm2324:sm2324

Learning objective

In this problem, we will explore the derivation of thermodynamic quantities for a simple system (an ideal paramagnet) using the formalism of the microcanonical ensemble.

For a microcanonical ensemble, we consider a perfectly isolated large system, made out of N independent magnetic moments of total energy E. Through the application of a homogeneous magnetic field H, each moment m_i will take one of two values, $m_i = \pm m$. The Hamiltonian $\mathcal{H}$ for the system is

$$\mathcal{H} = -\sum_{i=1}^{N} H\, m_i = -H\, M = -H\, n\, m\,, \tag{1}$$

where $n = n_+ - n_-$ denotes the difference of positive and negative moments and $M = n\, m$ the magnetization.

a) The phase space for this system is discrete, why? $\Omega(M)$ is the number of states for a given magnetization, explicitly calculate and show, that it takes on the form **1**pt(s)

$$\Omega(n) = \frac{N!}{\left[\frac{1}{2}(N+n)\right]!\ \left[\frac{1}{2}(N-n)\right]!}\,.$$

b) Compute the entropy $S(E,H)$ for the system with $S(E,H) = k_\mathrm{B}\log[\Omega(E,\ H)]$, neglect terms of order $\mathcal{O}(\log N)$ and smaller. **1**pt(s)

c) Determine the temperature $\frac{1}{T} = \frac{\partial S}{\partial E}\big|_H$ and afterwards solve it for $E = E(T,H)$. **1**pt(s)

d) Calculate the magnetization $M = T\,\frac{\partial S}{\partial H}\big|_E$ and explicitly demonstrate, that in the high temperature limit ($k_\mathrm{B}T \gg H\,m$) the Curie behavior $M = N\frac{Hm^2}{k_\mathrm{B}T}$ can be obtained. In addition, compute the isothermal magnetic susceptibility $\chi_T = \frac{\partial M}{\partial H}\big|_T$. **1**pt(s)

Problem 11.2: Entropy of a Simple System [**Oral** | 3 pt(s)]

ID: ex_entropy_of_a_simple_system_sm2324:sm2324

Learning objective

Here, we will try to grasp a straightforward and plausible interpretation of entropy within the framework of statistical mechanics.

Consider a simple magnetic lattice system with N spin-1 atoms. Each atom can be in one of three spin states, namely $S_z = \pm 1, 0$. The respective number of atoms in each of those spin states is denoted by $n_{\pm 1}$, n_0. No magnetic field shall be present, hence all states are degenerate.

a) Determine the total entropy of the system as a function of $n_{\pm 1}$ and n_0. For sake of convenience, use Stirling's formula. **1**pt(s)

b) Find the configuration (n_{-1}, n_0, n_1) which maximizes the entropy. How can the entropy be understood? **1**pt(s)

c) Calculate the entropy for the maximizing configuration. **1**pt(s)

Problem 11.3: Ideal Paramagnet Part 2

[**Oral** | 3 pt(s)]

ID: ex_ideal_paramagnet_part_2_sm2324:sm2324

Learning objective

In this exercise, we will once more determine the properties of an ideal paramagnet, similar to the previous problem 11.1. This exercise aims to rederive the thermodynamic quantities within the context of the canonical ensemble formalism. Upon completing the calculation, it will become apparent that approaching the problem through the canonical ensemble framework simplifies the analysis.

Here, we again consider an ideal paramagnet, as investigated in problem 1 from above. In this scenario, the analysis takes place in a canonical ensemble. Calculate the following quantities for the canonical ensemble:

a) Internal energy $E(T, H, N)$ **1**pt(s)

b) Entropy $S(T, H, N)$ **1**pt(s)

c) Magnetization $M(T, H, N)$ and isothermal magnetic susceptibility χ_T . Compare these two quantities with the microcanonical scenario of problem 1. **1**pt(s)