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WS 2023/24

Information on lecture and tutorials

Here a few infos on the modalities of the course ***"Theo IV: Statistische Mechanik"***:

- The `C@MPUS-ID` of this course in `042040011`.
- You can find detailed information on lecture and tutorials on the website of our institute:
  `https://itp3.info/sm2324`
- In addition, you can find information on lecture and tutorials on `ILIAS`:
  `https://ilias3.uni-stuttgart.de/goto_Uni_Stuttgart_crs_3409212.html`
- **Written** problems have to be handed in and will be corrected by the tutors. You must earn at least **80 %** of the written points to be admitted to the exam.
- **Oral** problems have to be prepared for the exercise session and will be presented by a student at the blackboard. You must earn at least **66 %** of the oral points to be admitted to the exam.
- Every student is required to **present** at least **2** of the oral problems at the blackboard to be admitted to the exam.
- Problems marked with an asterisk ($*$) are optional and can earn you bonus points.
- If you have questions regarding the problem sets, feel free to contact to your tutor at any time.

Problem 1.1: Partial Derivative

[**Written** | 5 pt(s)]

ID: ex_partial_derivative:sm2324

Learning objective

Thermodynamics deals with systems characterized by continuous functions of many variables. Here, partial derivatives help us to understand how a system's properties change with respect to change in the specific variables while keeping others variable constant. In this problem, you will revise the basics of partial derivatives and learn useful formulas associated with them.

The variables x, y and z are connected through $f(x, y, z) = 0$. There is a function $w(x, y)$ with two out of the three variables. Show that

a) $\left.\frac{\partial x}{\partial y}\right|_z = \left(\left.\frac{\partial y}{\partial x}\right|_z\right)^{-1}$ **1**pt(s)

b) $\left.\frac{\partial x}{\partial y}\right|_z \left.\frac{\partial y}{\partial z}\right|_x \left.\frac{\partial z}{\partial x}\right|_y = -1$ **1**pt(s)

c) $\left.\frac{\partial x}{\partial w}\right|_z = \left.\frac{\partial x}{\partial y}\right|_z \left.\frac{\partial y}{\partial w}\right|_z$ **1**pt(s)

d) $\left.\frac{\partial x}{\partial z}\right|_w = \left.\frac{\partial x}{\partial y}\right|_w \left.\frac{\partial y}{\partial z}\right|_w$ **1**pt(s)

e) $\left.\frac{\partial x}{\partial y}\right|_z = \left.\frac{\partial x}{\partial y}\right|_w + \left.\frac{\partial x}{\partial w}\right|_y \left.\frac{\partial w}{\partial y}\right|_z .$ **1**pt(s)

Problem 1.2: State variables

[**Oral** | 5 pt(s)]

ID: ex_state_variables:sm2324

Learning objective

In this problem you will learn about the state variables and their importance in the thermodynamics. As an example you will calculate the integrating factor to obtain an exact differential from an non-exact differential function.

It is known, that through small deviations dx, dy of the external parameters x, y the quantity E changes as

$$\delta E = F_x\, dx + F_y\, dy \,,$$

with the vector $\mathbf{F}(x,y) = [F_x(x,y)\,, F_y(x,y)]$. The quantity E is called a state variable, if δE is represented through an exact differential

$$dE = \partial_x\, E(x,y)\; dx + \partial_y\, E(x,y)\; dy \,.$$

a) Consider δE (i.e. $\mathbf{F}$), show the equivalence of the following two statements **1**pt(s)
   i. E is a state variable, i.e. $\exists E(x,y):\ \mathbf{F} = \nabla E$ and
   ii. $\nabla \wedge \mathbf{F} = 0$.

b) Why is E called a state variable? **1**pt(s)

c) Why are state variables important in thermodynamics? **1**pt(s)

d) If a differential δE is not exact, it is possible to find an integrating factor $\mu(x,y)$, such that $dS = \mu(x,y)\,\delta E$ becomes exact. Determine the integrating factor $\mu(x,y)$ for **1**pt(s)

$$\delta E = \left(xy^2 + xye^x\right) dx + \left(2x^2y + xe^x\right) dy \,,$$

under the assumption, that μ only depends on x. In addition determine $S(x,y)$.

e) Give an example for an exact differential, a non-exact differential and its integrating factor. **1**pt(s)

Problem 1.3: States of equilibrium

[**Oral** | 3 pt(s)]

ID: ex_states_of_equilibrium:sm2324

Learning objective

In this exercise, you will calculate the equilibrium state of a gas which is separated by a piston. You will learn how the equilibrium state of the gas changes if piston becomes diathermic and can move freely.

A hollow cylinder is separated into two chambers through a piston, see figure 1. The walls of the hollow cylinder shall be isolating. In the initial state, the piston is fixed and also isolating, whereas the chambers (1) and (2) have the volumes V_1 and V_2 and contain the number of molecules N_1 and N_2 of the same gas (e.g. helium), respectively. The pressure in this state is P_1 and P_2, respectively.

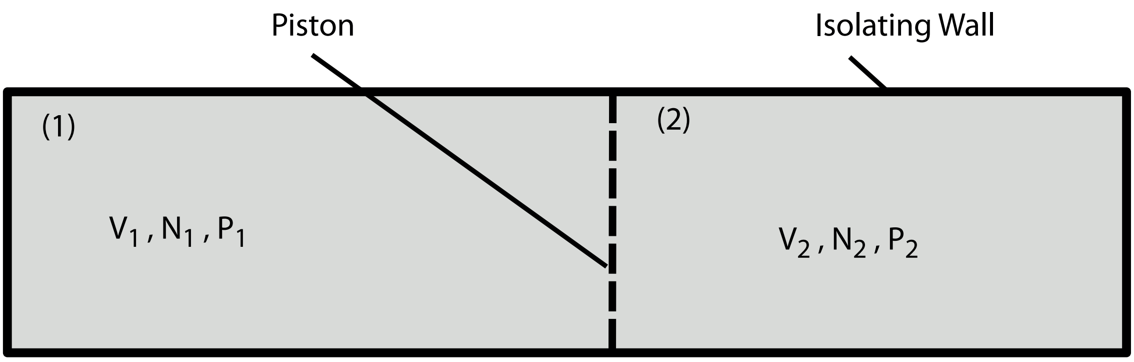

Abbildung 1: Hollow cylinder, separated into two chambers through a piston.

a) Now the piston shall be diathermic. What is the new state of equilibrium? **1** pt(s)

b) What is the new state of equilibrium, if the diathermic piston is able to move freely? **1** pt(s)

c) Does the state of equilibrium of problem b) change, if, in addition, the piston becomes permeable for the molecules of the gas? **1** pt(s)