↓ **Lecture 1** [17.10.23]

Setting the Stage

Terminology

The most important terms in this course and their German correspondence:

RELATIVITY = *Relativitätstheorie*
SPECIAL RELATIVITY = *Spezielle Relativitätstheorie (SRT)*
GENERAL RELATIVITY = *Allgemeine Relativitätstheorie (ART)*

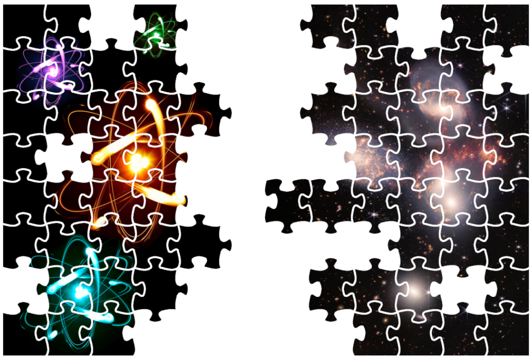
Relation of the theories:

RELATIVITY { SPECIAL RELATIVITY, GENERAL RELATIVITY }

Motivation

RELATIVITY is arguably the most popular of scientific theories, for it speaks about an entity of every day experience: space and time. This popularity comes with a caveat:

The "Mona Lisa perspective"
The popular status of RELATIVITY in physics parallels that of the Mona Lisa in arts: Einstein's magnum opus inherits an aura of perfection and finality.



The "Puzzle Perspective"
RELATIVITY is interesting because it describes *some*, but *not all* facets of reality. Its incompatibility with quantum mechanics hints at a reality even stranger than its pieces.

¡! You should *not* view RELATIVITY as the "Mona Lisa of physics" but as the harbinger of quantum gravity[1] that, most likely, will come with a reformulation of reality so profound that the "strangeness" of quantum mechanics and RELATIVITY alike will pale in comparison (→ *Excursions*).

[1] I use the term "quantum gravity" here very loosely and essentially synonymous with "theory of everything".

Ontology

1 | The ⁂ *ontology of physics* is the collection of "things that exist" (⁂ *entities*):

$$\text{Ontology} = \{ \underbrace{\underbrace{\text{Leptons, Hadrons, Higgs,}}_{\text{Matter: Atoms}\dots} \quad \underbrace{\text{Gauge bosons}}_{\text{Interactions: Photons}\dots}}_{\text{Standard Model of Particle Physics}} \}$$

2 | Physical theories are *models* that describe how these entities behave.

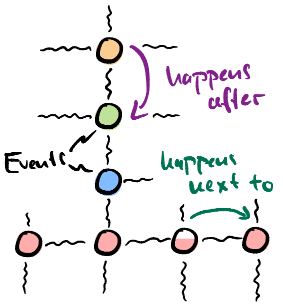
Examples:

Classical mechanics describes the dynamics *of* matter on macroscopic scales.
Quantum mechanics describes the dynamics *of* matter on microscopic scales.
Electrodynamics describes the dynamics *of* electromagnetic fields on macroscopic scales.

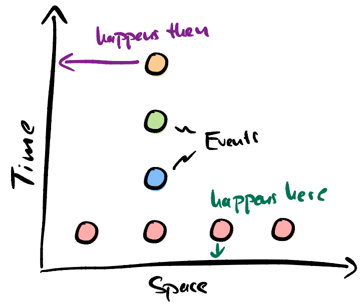
Note that these can be *effective* (approximate) descriptions that are restricted to finite scales of validity (length, energy, time).

3 | What is RELATIVITY a theory *of*?

i | ◁ Two notions of space and time:

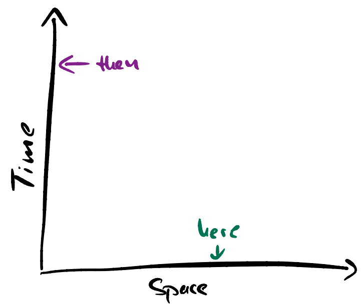


⁂ *Relational space & time*



⁂ *Newtonian space & time*

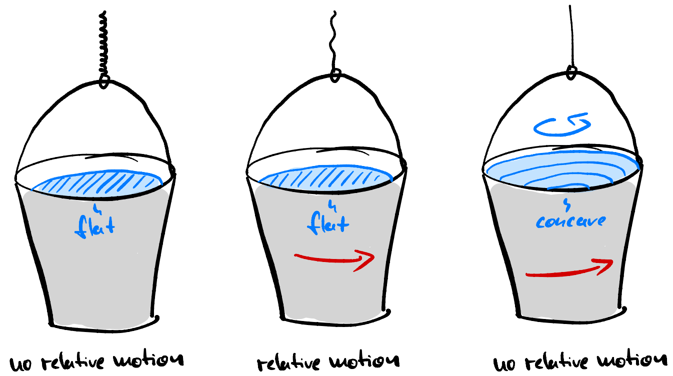
ii | ◁ Delete all entities from the world:



Nothing! Newtonian space & time left!

Question: Which notion describes reality?

iii | ◁ Newton's bucket:



Question: Rotation with respect to *what* determines the shape of the water surface?

Tentative answer: Rotation with respect to Newtonian space!

¡! Today, Newtonian space & time (sometimes called neo-Newtonian or Galilean spacetime) is *not* seen as a preferred ("absolute") coordinate system, with respect to which absolute *positions*, *times* and *velocities* can be measured; it is the entity that is responsible for the absolute notion of *acceleration* in Newtonian physics (which is also present in RELATIVITY). It is "the thing" that determines the reference frames that are *inertial* [5].

→ *Space & time (Spacetime)* is an independent "thing that exists."

The correct answer to the bucket experiment in RELATIVITY will be: The rotation with respect to the *local inertial frame*—which is determined by the local gravitational field—determines the shape of the water surface. This field is determined by the large-scale distribution of mass and energy in the universe, i.e., the fixed stars; the (rotating) mass of the earth has a non-zero but tiny effect as well (→ *Frame dragging*).

4 | Thus we should extend our ontology:

$$\text{Extended Ontology} = \{\underbrace{\underbrace{\text{Leptons},\ \text{Hadrons},\ \text{Gauge bosons},\ \text{Higgs}}_{\text{Standard Model of Particle Physics}},\ \underbrace{\textbf{Spacetime}}_{\text{RELATIVITY}}}_{\text{Core Theory} \xleftarrow{\text{IR}\quad \text{Energy scale}\quad \text{UV}} \text{Theory of Everything (?)}}\}$$

The ⁂ *Core Theory* [16] (→ *below*) is an effective (quantum) field theory that encompasses the standard model and RELATIVITY. It describes all entities know to us *on our scales*—but is expected to fail on the Planck scale (in the "UV limit"). The theory that the Core Theory renormalized to in this UV limit is the famous "Theory of Everything". This is uncharted territory and we do not know what this theory looks like.

The extended ontology above is known as ⁂ *substantivalism* in the philosophy of science, see [17] for a review and [18] for a supportive account of this ontology. Opposing substantivalism is ⁂ *relationalism*, which defends the view that spacetime is not an independent entity but an emergent description of *relations* between entitites (↑ *The Hole Argument*). Relationalism is exemplified by ↑ *Mach's principle*, which has been historically influential in the development of GENERAL RELATIVITY (though Einstein later changed his views). In the light of non-trivial solutions (of the Einstein field equations) for "empty" universes in GENERAL RELATIVITY, and the (now experimentally confirmed) existence of gravitational waves, I take a substantivalist stance in this course.

5 | This extended ontology allows us to answers the question:

> RELATIVITY is the theory *of* spacetime (on macroscopic scales), just as electrodynamics is the theory *of* the electromagnetic field.

¡! Despite these conceptual similarities, there is a fundamental difference between RELATIVITY and electrodynamics (→ *below*): Whereas electrodynamics describes the dynamics of the electromagnetic field *on* spacetime, the gravitational field of RELATIVITY does *not* evolve *on* spacetime; it *is* spacetime!

‡ The Core Theory

The ⁂ *Core Theory* S_* is the ⁂ *effective field theory* that describes all entities on the energy scales relevant for our everyday life [16]. As typical for a field theory, it is best expressed as a ↑ *path integral*:

$$A_* = \int_{k<\Lambda} \mathcal{D}g\, \mathcal{D}G\, \mathcal{D}\psi\, \mathcal{D}\phi\, \exp\left(\frac{i}{\hbar} S_*[g,G,\psi,\phi]\right).$$

$k<\Lambda$: Momentum cutoff; $\mathcal{D}g$: Gravitational field (Metric); $\mathcal{D}G$: Gauge fields (Photons...); $\mathcal{D}\psi$: Fermion fields (Electrons...); $\mathcal{D}\phi$: Higgs field (Higgs boson); S_*: Action

What makes this an *effective* theory is the momentum cutoff Λ: The theory describes the dynamics of the fields only up to some finite momentum/energy cutoff Λ. In [16] it is argued that $\Lambda \sim 10^{11}$ eV is a reasonable cutoff; since this is well below the Planck scale of 10^{28} eV, A_* does not describe the physics on these energy scales (e.g., what happens in black holes or near the Big Bang is *not* encoded in A_*). This reflects the lack of a consistent theory of quantum gravity.

The action S_* splits into two parts (plus one additional, technical term that we can savely ignore here):

$$S_*[g,G,\psi,\phi] = S_{\text{EH}}[g] + S_{\text{SM}}[g,G,\psi,\phi]\,.$$

The first part is the famous ⁂ *Einstein-Hilbert action* (G is the gravitational constant) and describes the gravitational field g:

$$S_{\text{EH}}[g] = \frac{1}{16\pi G}\int \mathrm{d}^4x\, \sqrt{-g}\, R(g)\,.$$

We will encounter this action in the second part of this course as it encodes the (source-free) ⁂ *Einstein field equations*; there you will learn what $R(g)$ is.

The second part is the action of the ⁂ *standard model of particle physics* (coupled to gravity via g) and describes all the stuff in our world (matter and interactions) *except gravity*:

$$S_{\text{SM}}[g,G,\psi,\phi] = \int \mathrm{d}^4x\, \sqrt{-g}\left[i\bar\psi \not{D}\psi - \frac{1}{4}G^2 + |D\phi|^2 - V(\phi) + \left(\bar\psi_L^i Y_{ij}\phi\psi_R^j + \text{h.c.}\right)\right].$$

$i\bar\psi \not{D}\psi$: Dirac (Fermion ke&i); $-\frac{1}{4}G^2$: Yang-Mills (Gauge boson ke&i); $|D\phi|^2$: Klein-Gordon (Higgs boson ke&i); $V(\phi)$: Higgs potential (Symmetry breaking); $\bar\psi_L^i Y_{ij}\phi\psi_R^j$: Yukawa coupling (Fermion masses)

Here "ke&i" stands for *kinetic energy* and *interactions* (with gauge bosons). The standard model action $S_{\text{SM}}[G, \psi, \phi] \equiv S_{\text{SM}}[\eta, G, \psi, \phi]$ on a static, flat spacetime $g = \eta$ is typically discussed in a course on quantum field theory with focus on high energy physics (↑ *Section 10.2 of my script on QFT [19]*). In this course on RELATIVITY, the existence of S_{SM} will leave its (classical) mark on the Einstein field equations in form of the ⁂ *energy-momentum tensor*.

Relation to other theories

1 | RELATIVITY is similar to other theories in that it is a theory *of* an entity that makes up reality. However, it is also different in that this very entity makes an appearance in most other theories:

Classical mechanics describes the macr. dynamics of **matter** *on* **spacetime**: $\vec{x}(t)$.
Quantum mechanics describes the micr. dynamics of **matter** *on* **spacetime**: $\Psi(\vec{x}, t)$.
Electrodynamics describes the macr. dynamics of **EM fields** *on* **spacetime**: $\boldsymbol{E}(\vec{x}, t)$, $\boldsymbol{B}(\vec{x}, t)$.

In the light of the extended ontology (where spacetime is an idependent entity described by RELATIVITY), it can be useful to reframe the objective of various theories as follows:

Classical mechanics describes the macr. dynamics of **matter** *interacting with* a (static) **spacetime**.
Quantum mechanics describes the micr. dynamics of **matter** *interacting with* a (static) **spacetime**.
Electrodynamics describes the macr. dynamics of **EM fields** *interacting with* a (static) **spacetime**.

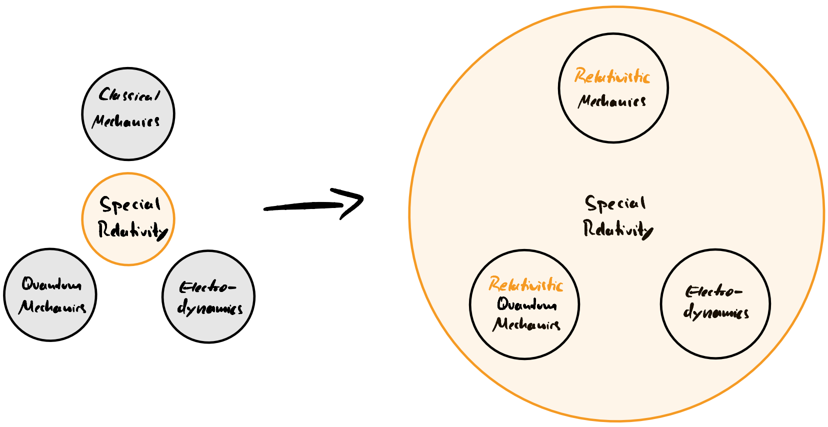
Note that this reading is manifest in the background-independent formulation of the Core Theory $S_\star[g, G, \psi, \phi]$ where the metric g and the other fields are treated on the same footing.

→ The properties of spacetime (as posited by RELATIVITY) must be reflected by these theories!

This means that we might have to *modify* known theories to be consistent with RELATIVITY. These modifications must adhere to the ⁂ *correspondence priciple*: The "old" (non-relativistic) versions of the theories must be included in the "new" (relativistic) versions as limiting cases.

2 | Incorporating the tenets of SPECIAL RELATIVITY leads to ...

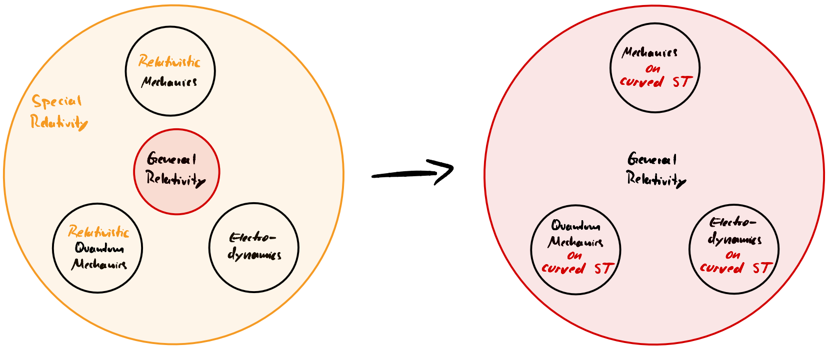
- *Relativistic* mechanics
- *Relativistic* quantum mechanics (Dirac equation, Klein-Gordon equation)
- *Relativistic* electrodynamics (= classical electrodynamics)



Luckily, classical electrodynamics is already consistent with SPECIAL RELATIVITY and needs no modification. By constrast, both classical mechanics and the quantum mechanics you learned in your previous courses must be *modified* to reflect the symmetries of spacetime posited by SPECIAL RELATIVITY.

3 | Incorporating the tenets of GENERAL RELATIVITY leads to ...

- (Relativistic) Mechanics *on curved spacetimes*
- (Relativistic) Quantum mechanics *on curved spacetimes*
- (Relativistic) Electrodynamics *on curved spacetimes*



In this course, we will discuss the modifications needed for *mechanics* and *electrodynamics* to fit the framework of GENERAL RELATIVITY. We won't discuss quantum mechanics on curved spacetimes.

¡! Quantum mechanics (describing matter and gauge bosons) on a curved spacetime is *not* "quantum gravity!" Quantum gravity is a theory where the metric field g *itself* is quantized (which we do not know how to do).

Spoiler

The gist of RELATIVITY can be summarized as follows:

$$\begin{aligned} \text{Spacetime} &\leftrightarrow \text{Four dimensional Lorentzian manifold } (M, g) \\ \text{Gravitational field} &\leftrightarrow \text{Metric tensor field } g \end{aligned}$$

This is what is meant by the popular statement that gravity "is not a force" but a geometrical deformation ("curvature") of spacetime.

and

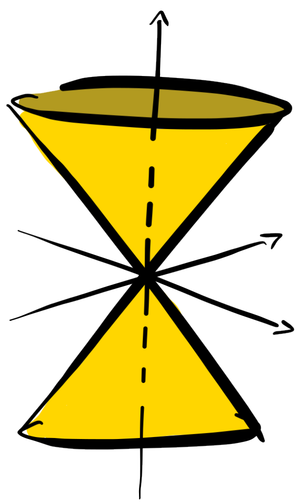
$$\begin{aligned} \text{SPECIAL RELATIVITY}: &\ g \text{ has signature } (1,3)\ (\textit{Lorentz symmetry}) \\ \text{GENERAL RELATIVITY}: &\ g \text{ is a dynamical field } (\textit{Background independence}) \end{aligned}$$

You most likely do not understand these statements at this point. That's fine! To provide you with the background knowledge to do so is the purpose of this course.

So let's start ...

Part I.

Special Relativity



1. Conceptual Foundations

◊ **Concepts**

- Events, Observations, Coincidences, Observers, Reference frames, Einstein synchronization, Cartesian coordinates, Inertial frames, Inertial coordinate systems, Coordinate transformations, Laws of nature, Physical models and theories
- Newtonian mechanics, Form-invariance and covariance, Invariance group, Active and passive transformations, Galilei transformations, Galilei group, Galilean principle of relativity
- Maxwell equations, Aether, Michelson Morley experiment, Principle of Special Relativity
- Isotropy, Homogeneity, Affine transformations
- Special Lorentz transformations, Lorentz Boosts, Lorentz group, Lorentz factor, Limiting velocity, Lorentz covariance, Addition of collinear velocities, Finite speed of causality
- Spacetime interval, Invariant interval, Time-like, Space-like, Light-like, Light cone, Invariant hyperbolae, Causality, Time-like trajectories, Partial order of events, Causal automorphism
- Relativity principles, Symmetries of spacetime, Simplicity of nature, Compressibility, Anthropic principle

1.1. Events, frames, laws, and models

1 | Events:

i | A. Einstein writes in his 1905 paper "Zur Elektrodynamik bewegter Körper" [9]:

> *Wir haben zu berücksichtigen, daß alle unserer Urteile, in welchen die Zeit eine Rolle spielt, immer Urteile über gleichzeitige Ereignisse sind. Wenn ich z. B. sage: "Jener Zug kommt hier um 7 Uhr an," so heißt dies etwa: "Das Zeigen des kleinen Zeigers meiner Uhr auf 7 und das Ankommen des Zuges sind gleichzeitige Ereignisse."*

And in his 1916 review "Die Grundlage der allgemeinen Relativitätstheorie" [20]:

> *Alle unsere zeiträumlichen Konstatierungen laufen stets auf die Bestimmung zeiträumlicher Koinzidenzen hinaus. Bestände beispielsweise das Geschehen nur in der Bewegung materieller Punkte, so wäre letzten Endes nichts beobachtbar als die Begegnungen zweier oder mehrerer dieser Punkte. Auch die Ergebnisse unserer Messungen sind nichts anderes als die Konstatierung derartiger Begegnungen materieller Punkte unserer Maßstäbe mit anderen materiellen Punkten bzw. Koinzidenzen zwischen Uhrzeigern, Zifferblattpunkten und ins Auge gefaßten, am gleichen Orte und zur gleichen Zeit stattfindenden Punktereignissen.*

We condense this into the following postulate:

§ Postulate: Invariance of coincidence `IC`

- Observations are *coincidences* of *events* local in space and time.
- Coincidences of events are absolute and observer independent.

ii | Example:

$$\begin{aligned} \text{Event } e_1 &= \langle \text{Clock A shows time 11:30} \rangle \\ \text{Event } e_2 &= \langle \text{Detector B detects electron} \rangle \\ \text{Event } e_3 &= \langle \text{Clock C shows time 9:45} \rangle \end{aligned}$$

If detector B and clock A are at the same location (spatial coincidence), and clock A shows 11:30 when detector B detects and electron (temporal coincidence), we say that the events e_1 and e_2 coincide: $e_1 \sim e_2$.

→ Collect all events e_i that coincide into an equivalence class E:

$$e_1 \sim e_2 \sim e_3 \sim \ldots \;\rightarrow\; E = \{e_1, e_2, e_3, \ldots\}$$

In a slight abuse of nomenclature we call the coincidence class E also *event*.

Note that this abuse of nomenclature is also used in everyday life: What makes up an "event" (like a party) is the set of all "little events" (like you meeting your friend) that happen (roughly) at the same location and the same time.

iii | Assumption:

The set $\mathcal{E} = \{E_1, E_2, \ldots\}$ of all coincidence classes is a complete, observer independent record of reality.

We call the information stored in $\mathcal{E}$ *absolute* because all observers agree on it.

2 | ⁂ *Observer $\mathcal{O}$ ≡ ⁂ (Reference) Frame $\mathcal{O}$*:

Goal: Systematic description of physical phenomena in terms of models.

Question: How to systematically observe reality and encode these observations?

:= Experimental setup to collect data about events in space & time: