Recap

10.2 The Standard Model

$$\boxed{\mathcal{L}_{SM} = \mathcal{L}_{EWS} + \mathcal{L}_{QCD} \quad (+ \ldots)}$$

$$\boxed{\mathcal{L}_{Fermion} + \mathcal{L}_{YM} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}}$$

✓ ✓ ✓

today

today

- Higgs SSB $\rightarrow \langle \Phi \rangle = \Phi_0 = \frac{1}{2}\begin{pmatrix} 0 \\ v \end{pmatrix}$

Electric Charge: $\boxed{Q = T^3 + Y}$

With $Y(\Phi) = +\frac{1}{2} \Rightarrow e^{i\hat{Q}\alpha(x)}\Phi_0 = \Phi_0$

$\rightarrow$ unbroken symmetry in $SU(2)_L \times U(1)_Y$

$\rightarrow$ Gauge group $U(1)_Q$ of QED

- Higgs field in unitary gauge.

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

real Higgs scalar $\rightarrow$ Higgs boson

New gauge fields

	Mass	Electric Charge Q
$W^\pm_\mu := \frac{1}{\sqrt{2}}\left(W^1_\mu \mp i W^2_\mu\right)$	$m_W > 0$	± 1
$Z_\mu := \frac{1}{\sqrt{g^2+g'^2}}\left(g W^3_\mu - g' B_\mu\right)$	$m_Z > 0$	0
$A_\mu := \frac{1}{\sqrt{g^2+g'^2}}\left(g' W^3_\mu + g B_\mu\right)$	0	0

$\Downarrow$

$$\mathcal{L}_{Higgs} = \left(D^\mu_H \Phi\right)^\dagger \left(D^\mu_H \Phi\right) + \ldots$$

$$= \underbrace{\left(\frac{gv}{2}\right)^2}_{m_W^2} W^+_\mu W^-_\mu + \frac{1}{2}\underbrace{\left(\frac{v}{2}\right)^2 (g^2+g'^2)}_{m_Z^2} (Z_\mu)^2 + \ldots\ldots$$

no mass for A_μ!

$\in \mathcal{L}_{YM}$

W^+ A W^-

Hypercharge fixed by Q:

$$\boxed{Y = Q - T^3}$$

Q: known (exp) ; T^3: fixed by $SU(2)_L$ irrep

Masses for Fermions from Yukawa coupling:

$$-\gamma_e (\overline{\Phi_L}, \Phi)_i e_R + h.c. = -\gamma_e [\phi^+ \bar{\nu}_L e_R + \phi^0 \bar{e}_L e_R] + h.c. \overset{\text{Higgs}}{=} -\frac{\gamma_e v}{\sqrt{2}} (\bar{e}_L e_R + \bar{e}_R e_L)$$

$SU(2)_L$ $(\mathbb{C}^2)$; $SO^+(1,3)$ $(\mathbb{C}^4)$; m_e ; Dirac mass term

→ Works for all Fermions with RH fields (Neutrinos?!)

ii) Most general Yukawa coupling:

$$\boxed{\begin{aligned}\mathcal{L}_{Yukawa} = &-\Gamma^u_{mn} \bar{Q}^m_L \hat{\Phi} u^n_R - \Gamma^d_{mn} \bar{Q}^m_L \Phi d^n_R \\ &-\Gamma^l_{mn} \bar{L}^m_L \Phi l^n_R - \Gamma^\nu_{mn} \bar{L}^m_L \hat{\Phi} \nu^n_R\end{aligned}} + h.c.$$

- $m, n \in \{I, II, III\}$: fermion generations
- $x \in \{u, d, l, \nu\}$: fermion types

$l^I_R = e_R$, $l^{II}_R = \mu_R$, $u^I_R = u_R$, $u^{II}_R = c_R$

- Γ^x_{mn}: coupling constants

$\Gamma^l_{I,I} = \gamma_e$

$\bar{\nu}_{eL} \nu_{eR} + h.c.$; $\bar{\nu}_{\mu L} \nu_{eR}$; $\nu_{\mu R}$; $\nu_{\tau R}$

- Q^m_L, L^m_L: left-handed quark, lepton doublets

$\bar{Q}^I_L = (\bar{u}_L, \bar{d}_L)$, $\bar{L}^{II}_L = (\bar{\nu}_{\mu L}\ \bar{\mu}_L)$

- $\hat{\Phi}_i = \varepsilon^{ij} \Phi^*_j$: Higgs doublet with opposite Hypercharge $Y(\hat{\Phi}) = -\frac{1}{2}$

iii) Yukawa coupling ...

* ... generates mass for quarks, charged fermions
* ... cannot generate mass for neutrinos if RH neutrinos are missing (see ⚡)
* ... leads to generation-changing transitions of quarks
* ... generation changing transitions for neutrinos (if RH neutrinos)

Note I:

sterile neutrinos

$$\mathcal{L}_{\text{Majoran}} = -\frac{1}{2} m_R \left(\overline{\nu}_R \nu_R^c + h.c.\right)$$

$$\bar{\nu} \underbrace{P_L P_R}_{=0} \nu \qquad \nu_R^c = C_{\frac{1}{2}} \nu^* = -i\gamma^2 \nu^*$$

$$\begin{pmatrix} 0 & \cdot \\ \cdot & 0 \end{pmatrix}$$

$$\overline{\nu}_L \nu_R$$

$$\begin{array}{ccccc} Y & = & T^3 & - & Q \\ \uparrow & & \uparrow & & \uparrow \\ 0 & & 0 & & 0 \end{array}$$

$\nu_L \to m_1$

$$m_1 \cdot m_2 \sim m^2$$

Note II:

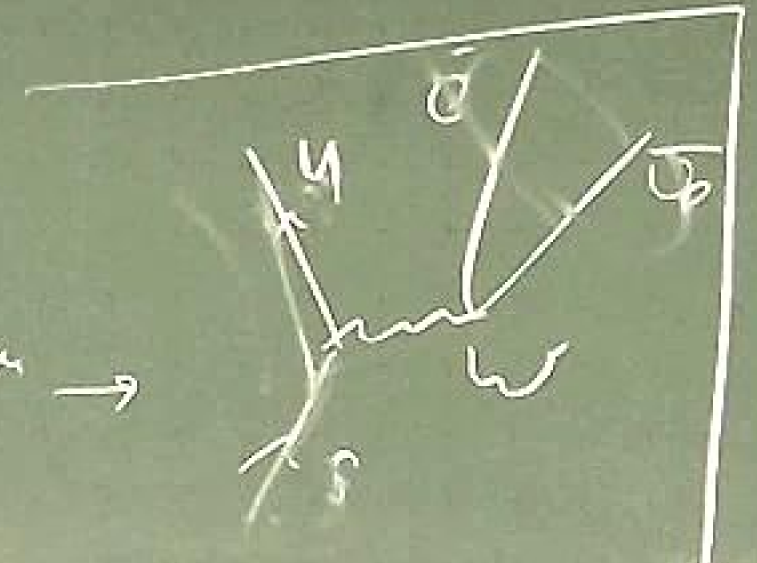
$$\Gamma^u_{mn} Q_L^{um} \hat{\Phi} u_R^n \rightarrow \text{CKM matrix}$$

$$Q_L \, \Gamma \, u_R$$

$$\underbrace{U^\dagger D V}_{} \quad \underline{V u_R}$$

$$\begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} \qquad \tilde{u} \qquad u = V^\dagger \tilde{u}$$

$$\underline{W \tilde{u} \tilde{d}} \rightarrow W^{um} \tilde{u}^u \tilde{d}^m \rightarrow$$



10.2.4 Quantum Chromodynamics

1) Gauge symmetry

$$\boxed{SU(3)_C} \quad \text{Color charge}$$

→ 8 generators $\qquad (\dim SU(N) = N^2-1)$

$Q^a,\ a = 1, \dots, 8$

$$[Q^a, Q^b] = i f^{abc} Q^c$$

→ Irreps:

- 1D: Trivial rep. $\hat{Q}^a = 0$ (Singlet)
- 3D rep: Defining representation

$$\hat{Q}^a = \frac{\lambda_a}{2} \quad \text{with } 3\times 3 \text{ Hermitian}$$

Gell-Mann matrices λ_a

(Triplet)

2) Field representations:

- Quarks = $SU(3)_C$ triplets

$$q = \begin{pmatrix} q_r \\ q_g \\ q_b \end{pmatrix} \quad \text{for } q \in \{u, d, c, s, t, b\}$$

$$\boxed{u_L \quad \mathbb{C}_Y \otimes \mathbb{C}^2_L \otimes \mathbb{C}^4_{SO^4} \otimes \mathbb{C}^3_c}$$

with colors r (red), g (green), b (blue)

- Leptons & Higgs: $SU(3)_C$ singlets

(Ignore in QCD)

→ Gauge trafo of Quark fields:

$$\tilde{q} = \underbrace{e^{i\hat{Q}^a \beta^a(x)}}_{U_C(x)} q$$

3) Lagrangian:

$$\boxed{\mathcal{L}_{QCD} = \sum_q \bar{q}(i \not{D}_c) q - \frac{1}{2}(G^a_{\mu\nu})^2}$$

3] $$\boxed{\mathcal{L}_{QCD} = \sum_q \bar{q}(i\not{D}_c)q - \frac{1}{4}(G^a_{\mu\nu})^2}$$

- $D_{c_\mu} = \partial_\mu - i g_s G^a_\mu \vec{\lambda}^a$
- g_s: coupling constant
- G^a_μ: 8 gauge fields → 8 Gluons
- $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu$

Gluons color charge:

Quark → $|r\rangle, |g\rangle, |b\rangle$

Gluon → $|r\rangle\langle r|$, $|r\rangle\langle g|$

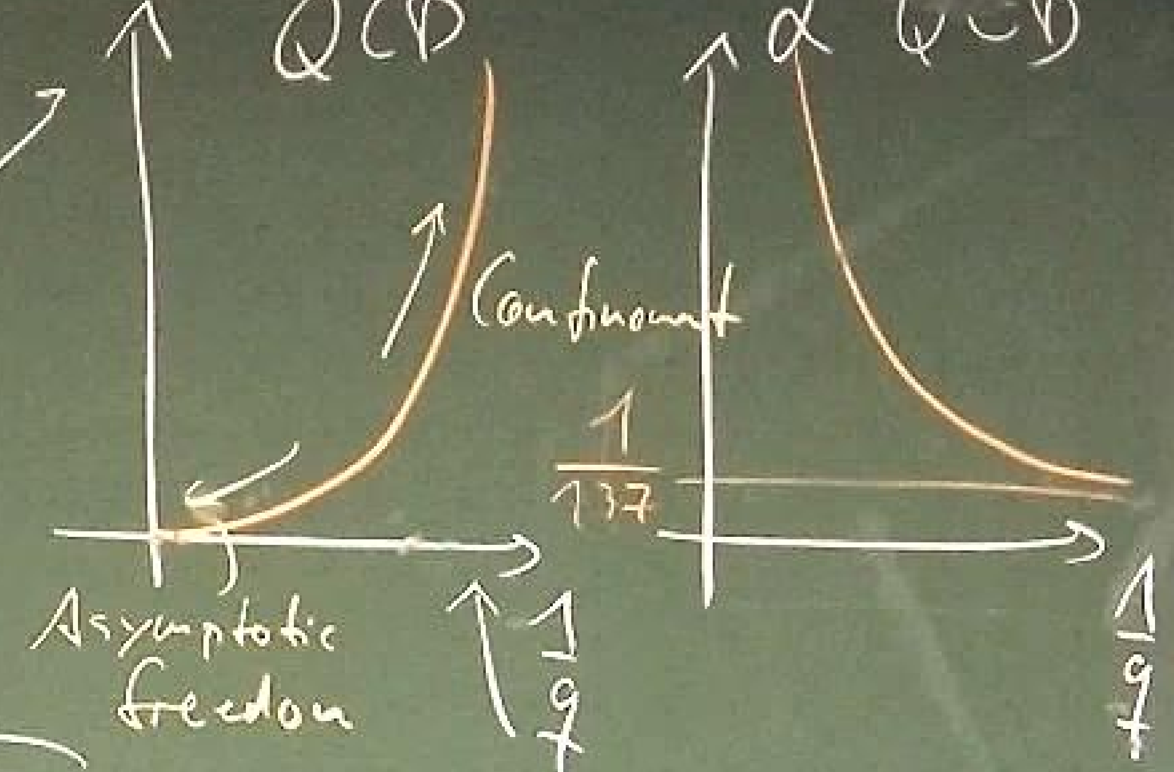
→ $|r\rangle\langle r| + |g\rangle\langle g| + |b\rangle\langle b|$ does not exist

→ 8 gluons

$SU(3) \leftrightarrow U(3)$

$$\alpha_s = \frac{g_s^2}{4\pi}$$

$\alpha_s^{eff}(q^2)$



10.2.5 Summary

$\underbrace{SU(2)_L \times U(1)_Y}_{\text{Higgs SSB}} \times SU(3)_C$

$U(1)_Q$

- [2 Lepton + 2 Quarks × 3 Colors] × 3 Generations = 24 Dirac spinors → $\boxed{96 \text{ complex fields}}$
- Parameters:
  - 9 × Fermions masses $m_e, m_u, \ldots$
  - 1 × Higgs mass $m_h \approx 125$ GeV
  - 1 × Higgs VEV v
  - 3 × Gauge field couplings g, g', g_s
  - 4 × CKM matrix parameters

18 Parameters

GUT ?

- $SU(5)$ ←
- $Spin(10) \sim SO(10)$ (y, d, r, g, b)

$SO(N)$ $Spin(N)$

$SO(3)$ $Spin(3) \simeq SU(2)$