## Recap

## 10.2. The Standard Model

## 10.2.3. The GWS-Theory

### 1) Lagrangian:

$$\boxed{\mathcal{L}_{EWS} = \mathcal{L}_{Fermion} + \mathcal{L}_{YM} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}}$$

Gauge Symmetry: $SU(2)_L \times U(1)_Y$

Weak Isospin (generators $\hat{T}^i$, $i = 1,2,3$) and Weak Hypercharge (generator $\hat{Y}$; short: $Y \cdot \mathbb{1}$)

Generators:

$$[\hat{T}^i, \hat{T}^j] = i\varepsilon^{ijk}\hat{T}^k, \quad [\hat{Y}, \hat{T}^i] = 0$$

Field Representations: ($SU(2)_L$)

LH Doublets: $\Psi_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \ldots$

$T^3(e_L) = -\frac{1}{2}, \quad T^3(\nu_{eL}) = +\frac{1}{2}, \ldots$

RH Singlets: $\psi_R = u_R, d_R, e_R, \ldots$ ← no RH neutrino!

$T^3(u_R) = 0, \ldots$

Higgs Doublet: $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

Hypercharge (irrep) still undefined! ($U(1)_Y$)

Gauge Trafos:

$$\boxed{\begin{aligned} \tilde{\Psi}_L &= e^{i\hat{Y}_L\alpha(x)} e^{i\hat{T}^j\beta^j(x)} \Psi_L \\ \tilde{\psi}_R &= e^{i\hat{Y}_R\alpha(x)} \psi_R \\ \tilde{\Phi} &= e^{i\hat{Y}_H\alpha(x)} \underbrace{e^{i\hat{T}^j\beta^j(x)}}_{V_L} \Phi \end{aligned}}$$

$$\boxed{\begin{aligned} \tilde{B}_\mu &= B_\mu + \frac{1}{g'}\partial_\mu\alpha \\ \tilde{W}_\mu &= V_L\left[W_\mu + \frac{i}{g}\partial_\mu\right]V_L^+ \end{aligned}}$$

Lagrangians:

$$\boxed{\mathcal{L}_{Fermion} = \sum_{\Psi_L} \bar{\Psi}_L(i\not{D}_L)\Psi_L + \sum_{\psi_R} \bar{\psi}_R(i\not{D}_R)\psi_R}$$

$\not{D}_L = \not{\partial} - ig\not{W}_L - ig'\not{B}_L$, with $\not{W}_L = \gamma^\mu W_\mu^i \hat{T}^i$, $\not{B}_L = \gamma^\mu B_\mu \hat{Y}_L$

$\not{D}_R = \not{\partial} - ig'\not{B}_R$

Dirac mass term not gauge invariant:

$SU(2)_L$ doublet → $\bar{\Psi}_L\psi_R$ ← $SU(2)_L$ singlet

$$\boxed{\mathcal{L}_{YM} = -\frac{1}{4}(B^{\mu\nu})^2 - \frac{1}{4}(W^i_{\mu\nu})^2}$$

$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\varepsilon^{ijk} W^j_\mu W^k_\nu$

$$\boxed{\mathcal{L}_{Higgs} = (D_H^{\mu}\Phi)^\dagger (D^{\mu}_H \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda(\Phi^\dagger\Phi)^2}$$

$SU(2)_L \times U(1)_Y$ gauge invariant

$$D_{H\mu} = \partial_\mu - i g \underbrace{W_\mu}_{W_\mu^i \hat{T}^i} - i g' \underbrace{B_\mu}_{B_\mu \hat{Y}_H}$$

8) Higgs mechanism I. Masses for gauge bosons

i) $\mu^2 < 0 \rightarrow$ VEV of Higgs field.

W.l.o.g. $\langle \Phi \rangle = \Phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$

$$v = \sqrt{\frac{-\mu^2}{\lambda}}$$

ii) Define electric charge operator:

$$\boxed{Q := T^3 + Y} \in su(2)_L \oplus u(1)_Y$$

$$SU(2)_L \times U(1)_Y$$

$\rightarrow$ Choose $Y(\Phi) = \boxed{+\frac{1}{2}}$ $\left( \hat{Y}_H = \frac{1}{2}\mathbb{1} \right)$

$$\hat{Q}\Phi_0 = (\hat{T}^3 + \hat{Y}_H)\Phi_0$$

$$= \left[ \frac{1}{2}\begin{pmatrix} 1 & \\ & -1 \end{pmatrix} + \boxed{\frac{1}{2}}\begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \right] \Phi_0$$

$$= \left( -\frac{1}{2} \boxed{+\frac{1}{2}} \right) \Phi_0 = 0$$

$$e^{i\hat{Q}\alpha(x)} \Phi_0 = e^0 \Phi_0 = \Phi_0$$

$\rightarrow$ Gauge symmetry $U(1)_Q$ generated by $Q$ is unbroken

$$\boxed{SU(2)_L \times U(1)_Y \xrightarrow{3\times SSB} \underbrace{U(1)_Q}_{\text{Unbroken gauge symmetry of QED}}}$$

Note: $\hat{Q}\begin{pmatrix} \phi^+ \\ 0 \end{pmatrix} = \left(\frac{1}{2} + \frac{1}{2}\right)\begin{pmatrix} \phi^+ \\ 0 \end{pmatrix} = 1 \begin{pmatrix} \phi^+ \\ 0 \end{pmatrix}$

$\hat{Q}\begin{pmatrix} 0 \\ \phi^0 \end{pmatrix} = \left(-\frac{1}{2} + \frac{1}{2}\right)\begin{pmatrix} 0 \\ \phi^0 \end{pmatrix} = 0 \begin{pmatrix} 0 \\ \phi^0 \end{pmatrix}$

iii] * Fluctuations of $\Phi$ around $\Phi_0$ in unitary gauge.

$$\Phi(x) \underset{\text{Unitary gauge}}{=} \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

$h(x)$: real scalar Higgs field

iv] $\Phi(x)$ in Lagrangian:

$$(D_H^{\mu}\Phi)^{\dagger}(D_{H\mu}\Phi) \doteq \frac{v^2}{8}\Big\{ g^2\big[(W_\mu^1)^2 + (W_\mu^2)^2\big] + \left(-g\, W_\mu^3 + g' B_\mu\right)^2 \Big\} + \ldots$$

v] Define new fields.

$$\boxed{\begin{aligned} W_\mu^{\pm} &= \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i\, W_\mu^2\right) \\ Z_\mu &= \frac{1}{\sqrt{g^2+g'^2}}\left(g W_\mu^3 - g' B_\mu\right) \\ A_\mu &= \frac{1}{\sqrt{g^2+g'^2}}\left(g' W_\mu^3 + g B_\mu\right) \end{aligned}}$$

$$\ominus\ \underbrace{\left(\frac{gv}{2}\right)^2}_{m_{W^\pm}^2} W_\mu^+ W^{-\mu} + \underbrace{\frac{1}{2}\left(\frac{v}{2}\right)^2 (g^2+g'^2)}_{m_Z^2} (Z_\mu)^2 + \ldots$$

$\phi^\dagger\phi$

$$\boxed{D_{H\mu} = \partial_\mu - (\;..\;) - i\underbrace{\frac{g g'}{\sqrt{g^2+g'^2}}}_{\text{Electric charge } e} A_\mu \hat{Q}}$$

- $A_\mu$: massless, neutral $(Q=0)$ gauge field $\rightarrow$ QED
- $W_\mu^{\pm}$: massive, charged $(Q=\pm 1)$ gauge bosons of weak interaction
- $Z_\mu$: massive, neutral $(Q=0)$ gauge boson of weak interaction

9) & Hypercharge

$Q = T^3 + Y$

- $Y(e_L) = Q(e_L) - T^3(e_L)$

$= -1 - (-\frac{1}{2}) = -\frac{1}{2}$

- $Y(e_R) = Q(e_R) - T^3(e_R)$

$= -1 - 0 = -1$

10) Higgs mechanism II: Masses for fermions

i) $\Psi_L^{i\alpha - 1,2} \quad \psi_R^{i - 0,1,2,3} \quad \Phi^a$

$\rightarrow -\gamma_e (\bar{\Psi}_L \Phi) e_R + h.c. = \gamma_e (\bar{\nu}_{eL}\ \bar{e}_L) \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + h.c.$

coupling constant

$\Psi_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$

$= \gamma_e \phi^+ \bar{\nu}_{eL} e_R + \gamma_e \phi^0 \cdot \bar{e}_L e_R + h.c.$

Higgs: $\phi^+ = 0$, $\phi^0 = \frac{v + h(x)}{\sqrt{2}}$

$\rightarrow$ Yukawa Coupling

$Y(\Phi) + Y(e_R) - Y(\Psi_L) = \frac{1}{2} - 1 - (-\frac{1}{2}) = 0$

Higgs SSB

$= -\frac{\gamma_e v}{\sqrt{2}} (\bar{e}_L e_R + \bar{e}_R e_L) + \dots$

$m_e = \frac{\gamma_e v}{\sqrt{2}}$