Recap

10.2 The Standard Model

10.2.1 Preliminaries

Chiral fermion fields.

$$\psi_R \equiv P_R \psi, \quad \psi_L \equiv P_L \psi$$

with $P_{R/L} = \frac{1}{2}(\mathbb{1} \pm \gamma^5) \overset{\text{Weyl}}{=} \begin{pmatrix} 0/1 & 0 \\ 0 & 1/0 \end{pmatrix}$

$\Downarrow$

$$\bar\psi(i\partial\!\!\!/ - m)\psi = \bar\psi_R(i\partial\!\!\!/)\psi_R + \bar\psi_L(i\partial\!\!\!/)\psi_L - m\bar\psi_L\psi_R - m\bar\psi_R\psi_L$$

Lorentz invariant

can transform differently under gauge groups!

10.2.2 Overview

Fields.

Fermions: (Spin-1/2)

have not been observed!

		I	II	III
Leptons	e_L e_R		μ ..	τ ..
	ν_{eL} (ν_{eR})		ν_μ ..	ν_τ ..
Quarks	u_L u_R		c ...	t ...
	d_L d_R		s ..	b ..

Generations

Chiral bispinor field

Vector Bosons (Spin-1)

Electroweak	Strong
• $SU(2)_L \times U(1)_Y$	• $SU(3)_C$
• $W_\mu^{i=1,2,3}, B_\mu$	• $G_\mu^{a=1,..,8}$
• $\underbrace{\gamma, W^+, W^-, Z}_{\text{after Higgs SSB}}$	• 8 Gluons

not gauge field of QED!

Scalar bosons (Spin-0)

$2\times$ complex Higgs field ϕ^+, ϕ^0

$\xrightarrow[\text{SSB}]{\text{Higgs}}$ $1\times$ real Higgs field h

Lagrangian:

Gauge fixing + Ghosts?

$$\mathcal{L}_{SM} = \mathcal{L}_{EWS} + \mathcal{L}_{QCD} \; (+ \quad)$$

$\boxed{\text{Electroweak Standard Model (Unification of EM and Weak force)}}$

Quantum Chromodynamics (Strong force)

Today

10.2.3 The Glashow-Weinberg-Salam Theory

$\mathcal{L}$ Goal: Generalize Higgs mechanism to SM

1) Lagrangian:

$$\boxed{\mathcal{L}_{EWS} = \mathcal{L}_{Fermion} + \mathcal{L}_{Yang\text{-}Mills} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}}$$

2) Gauge Symmetry:

$$\boxed{\underbrace{SU(2)_L}_{\text{Weak Isospin}} \times \underbrace{U(1)_Y}_{\text{Weak Hypercharge}}}$$

- $SU(2)_L \rightarrow$ 3 generators T^i, $i = 1,2,3$

$$[T^i, T^j] = i\varepsilon^{ijk} T^k$$

$\rightarrow$ Irreps:

- 1D. Trivial rep. $\hat{T}^i = 0$ (Singlet)
- 2D. Pauli matrices. $\boxed{\hat{T}^i = \frac{\sigma^i}{2}}$ (Doublet)

$\rightarrow$ Eigenvalue of $\hat{T}^3$ = The weak isospin T^3

($T^3 = \pm\frac{1}{2}$ for doublet, $T^3 = 0$ for singlet)

- $U(1)_Y \rightarrow$ 1 generator Y

$$[Y, T^i] = 0$$

$\rightarrow \hat{Y} =$ Number $\times \mathbb{1}$ = Hypercharge $\times \mathbb{1}$

3) $SU(2)_L$ Representation

- Left-handed fields = Isospin doublets

$$\Psi_L = \underbrace{\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}}_{\text{1st gen}}, \text{ 2nd, 3rd}$$

$\rightarrow$ Weak isospin. $T^3(\nu_{eL}) = +\frac{1}{2}$

$T^3(e_L) = -\frac{1}{2}$

Note: $\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$

$$\nu_{eL}(x) = \underbrace{\psi_L(x)}_{\in L^2 \otimes \mathbb{C}^4} \otimes \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{\mathbb{C}^2_L}$$

$$T^3(\nu_{eL}) = +\frac{1}{2} \Leftrightarrow +\frac{1}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\underline{T}^3 \nu_{eL} = \psi_L \otimes \overbrace{T_3 \begin{pmatrix} 1 \\ 0 \end{pmatrix}} = \frac{1}{2} \nu_{eL}$$

$$\frac{\sigma_3}{2} = \begin{pmatrix} \frac{1}{2} & \\ & -\frac{1}{2} \end{pmatrix}$$

- Right-handed fields = Isospin singlets

$$\psi_R = \underbrace{u_R, d_R, e_R}_{1\text{st. gen}}, \ 2\text{nd}, \ 3\text{rd}$$

$$\to T^3(e_R) = 0$$

- Higgs fields = Isospin doublet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \to T^3(\phi^+) = \frac{1}{2}$$

$$T^3(\phi^0) = -\frac{1}{2}$$

3) $SU(2)_L$ Representation

- Left-handed fields = Isospin doublets

$$\Psi_L = \underbrace{\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}}_{1\text{st. gen}}, \ 2\text{nd}, \ 3\text{rd}$$

$\to$ Weak isospin: $T^3(\nu_{eL}) = +\frac{1}{2}$

$T^3(e_L) = -\frac{1}{2}$

→ Gauge transformation on fields:

$$\boxed{\begin{aligned} &\text{LH-doublet:} && \tilde{\Psi}_L = e^{i\hat{Y}_L\alpha(x)} \underbrace{e^{i\hat{T}^i\beta^i(x)}}_{V_L} \Psi_L \\ &\text{RH-singlet:} && \tilde{\psi}_R = e^{i\hat{Y}_R\alpha(x)} \psi_R \\ &\text{Higgs doublet:} && \tilde{\Phi} = e^{i\hat{Y}_H\alpha(x)} e^{i\hat{T}^i\beta^i(x)} \Phi \end{aligned}}$$

where $\hat{Y}_L = Y\cdot\mathbb{1}$, $\hat{Y}_R = Y\cdot\mathbb{1}$, $\hat{Y}_H = Y\cdot\mathbb{1}$

Note: Y is fixed for each irrep.

$$Y(u_L) = Y(d_L), \quad Y(u_R) \neq Y(e_R)$$

4) Kinetic energy for fermions:
(& Minimal coupling)

$$\boxed{\mathcal{L}_{\text{Fermion}} = \sum_{\Psi_L} \bar{\Psi}_L i \not{D}_L \Psi_L + \sum_{\psi_R} \bar{\psi}_R i \not{D}_R \psi_R}$$

with cov. derivatives:

$$\boxed{\begin{aligned} D_{L\mu} &= \partial_\mu - i g W^i_\mu \hat{T}^i - i g' B_\mu \hat{Y}_L \\ D_{R\mu} &= \partial_\mu - i g' B_\mu \hat{Y}_R \end{aligned}}$$

coupling constants

→ Transformation of gauge fields

$$\boxed{\begin{aligned} \tilde{B}_\mu &= B_\mu + \frac{1}{g'}\partial_\mu\alpha \\ \tilde{W}_\mu &= V_L\left[W_\mu + \frac{i}{g}\partial_\mu\right]V_L^\dagger \end{aligned}}$$

$W^i_\mu \hat{T}^i$

Γ_{QED}

$\bar{\psi}(i\not{\partial} - m)\psi$

$= \bar{\psi}_L(i\not{\partial})\psi_L + \bar{\psi}_R(i\not{\partial})\psi_R + \dots$

$P = \gamma^0 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$

Example 10.1. β-Decay:

$$\mathcal{L}_{Fermion} = (\bar u_L\ \bar d_L)\, i\not{D}_L \begin{pmatrix}u_L\\ d_L\end{pmatrix} + (\bar \nu_{eL}\ \bar e_L)\, i\not{D}_L \begin{pmatrix}\nu_{eL}\\ e_L\end{pmatrix} + \dots$$

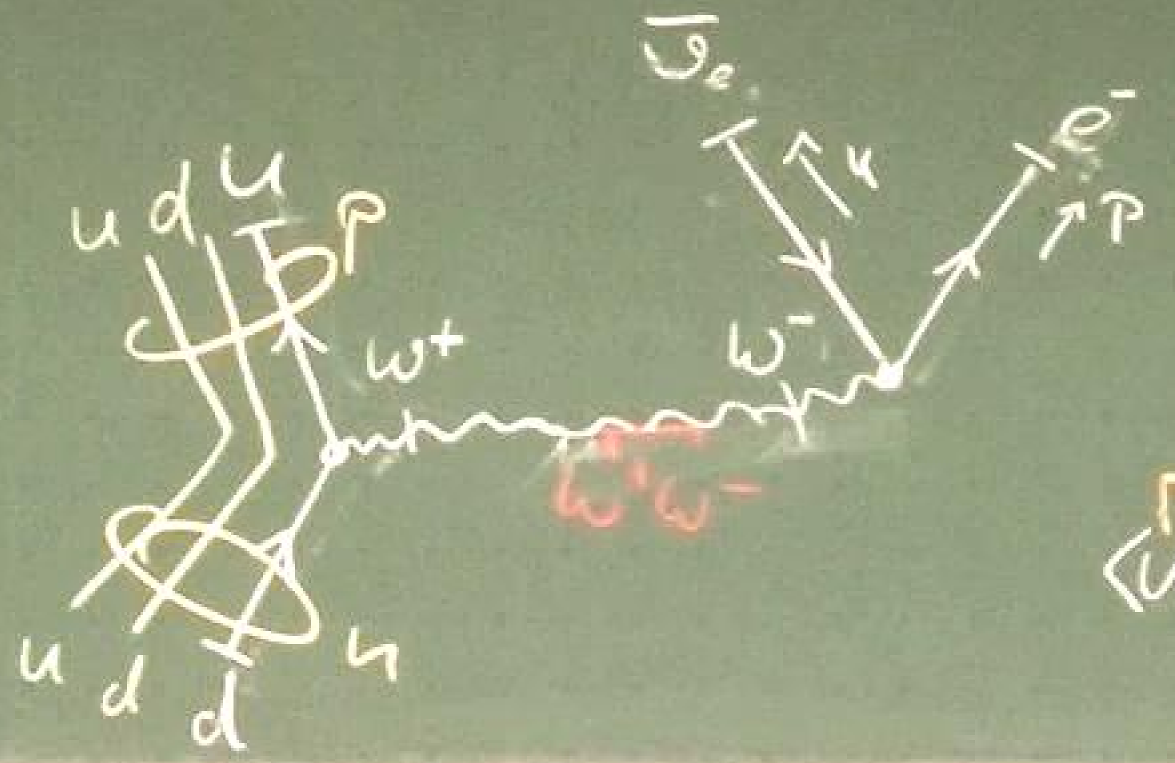
$$D_{L\mu} = \dots \underline{-ig(W^1_\mu \hat T^1 + W^2_\mu \hat T^2)} + \dots$$

$$= -i\frac{g}{\sqrt{2}}\begin{pmatrix}0 & W^1_\mu - iW^2_\mu\\ W^1_\mu + iW^2_\mu & 0\end{pmatrix} + \dots$$

$$\boxed{W^\pm_\mu := \frac{1}{\sqrt{2}}(W^1_\mu \mp i W^2_\mu)}$$

$$\boxed{\mathcal{L}_{Fermion} \sim W^+_\mu \bar u_L \gamma^\mu d_L + W^-_\mu \bar e_L \gamma^\mu \nu_{eL} + h.c.} \dots$$

$$\left[(\bar u_L\ \bar d_L)\begin{pmatrix}0 & W^+\\ W^- & 0\end{pmatrix}\begin{pmatrix}u_L\\ d_L\end{pmatrix} = (\dots)\begin{pmatrix}W^+ d_L\\ W^- u_L\end{pmatrix} = \bar u_L W^+ d_L + \bar d_L W^- u_L\right]$$



$\rightarrow$ β^--decay

$$\langle u, e, \bar\nu_e | W^+_\mu \bar u_L d_L W^-_\mu \bar e_L \nu_{eL} | d\rangle$$

5] Dirac mass term?

$m\left(\bar{\Psi}_L \psi_R + \bar{\psi}_R \Psi_L\right) \rightarrow$ Undefined

$\bar{\Psi}_L^a \psi_R$

→ $SU(2)_L$ invariant since

- $\bar{X}_L$ is component of an $SU(2)$ doublet
- Y_R is a $SU(2)$ singlet

→ $\bar{X}_L Y_R$ is not a $SU(2)$ singlet

→ We cannot add Dirac mass term!

6] Kinetic energy for gauge bosons.

(Yang-Mills-Lagrangian)

$$\mathcal{L}_{\text{Yang-Mills}} = -\frac{1}{4}\left(B^{\mu\nu}\right)^2 - \frac{1}{4}\left(W^i_{\mu\nu}\right)^2$$

Field strength: $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\,\varepsilon^{ijk} W^j_\mu W^k_\nu$$

7] Higgs field.

$$\mathcal{L}_{\text{Higgs}} = \left(D^\mu_H \Phi^\dagger\right)\left(D_{H\mu} \Phi\right) - \mu^2 \Phi^\dagger \Phi - \lambda\left(\Phi^\dagger \Phi\right)^2$$

Cov. deriv.:

$$D_{H\mu} = \partial_\mu - i g W^i_\mu \hat{T}^i - i g' B_\mu \hat{Y}_H$$

$$\left(\phi^\dagger \phi\right)^2 \neq |\phi^+|^4 + |\phi^0|^4$$