Recap

10. Excursions

10.1 The Higgs Mechanism

1| Maxwell + Complex scalar

$$\mathcal{L} = -\frac{1}{4}F^2 + |D\phi|^2 - V(\phi)$$

Potential. $V(\phi) = \mu^2|\phi|^2 + \lambda|\phi|^4$

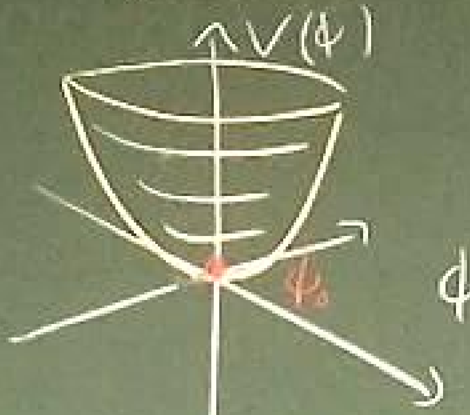
2| Gauge symmetry: $U(1)$

$$\tilde{\phi} = e^{i\alpha(x)}\phi$$

$$\tilde{A}_\mu = A_\mu - \frac{1}{e}\partial_\mu \alpha(x)$$

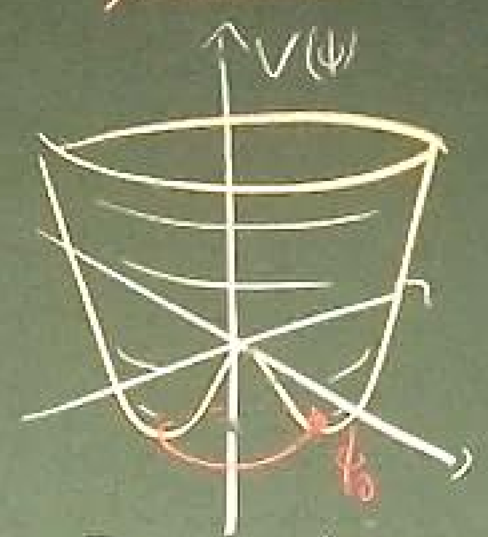
3| $\lambda > 0$, $\phi \in \mathbb{C}$

$\boxed{\mu^2 > 0}$



Unique minimum:
$\langle \phi \rangle = \phi_0 = 0$

$\boxed{\mu^2 < 0}$



Degenerate minima.

$v = |\phi_0| = |\langle \phi \rangle| = \sqrt{\frac{-\mu^2}{2\lambda}} \neq 0$

Vacuum expectation value (VEV)

→ Ground state breaks global $U(1)$ symmetry spontaneously

→ Spontaneous Symmetry Breaking

4| Goldstone Theorem.

Spontaneous breaking of global, continuous symmetry

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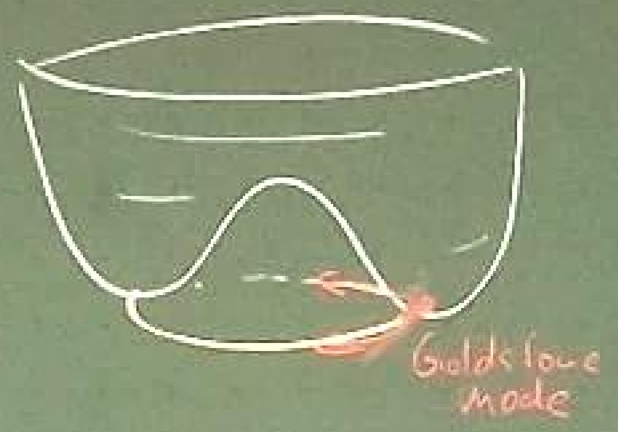
massless scalar particle in spectrum
(Nambu-Goldstone Boson)

"Proof":



Examples:

- Phonons ($O(3) \times \mathbb{R}^3$)
- Magnons ($SO(3)$)

Exception.

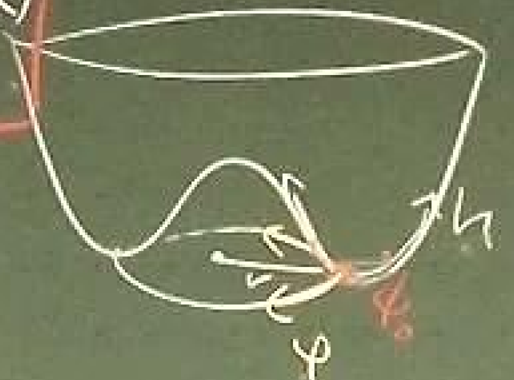
Superconductivity
(SSB of global $U(1)$ sym)
but no Goldstone mode

⇓

How can Goldstone Theorem fail?

5] ✗ $\langle\phi\rangle = \phi_0 = v$ breaks global $U(1)$ sym.

$$\rightarrow \boxed{\phi(x) = (v + h(x))\, e^{i\varphi(x)}}$$



2 real fields.

- $h(x)$. Higgs field
- $\varphi(x)$. Goldstone boson

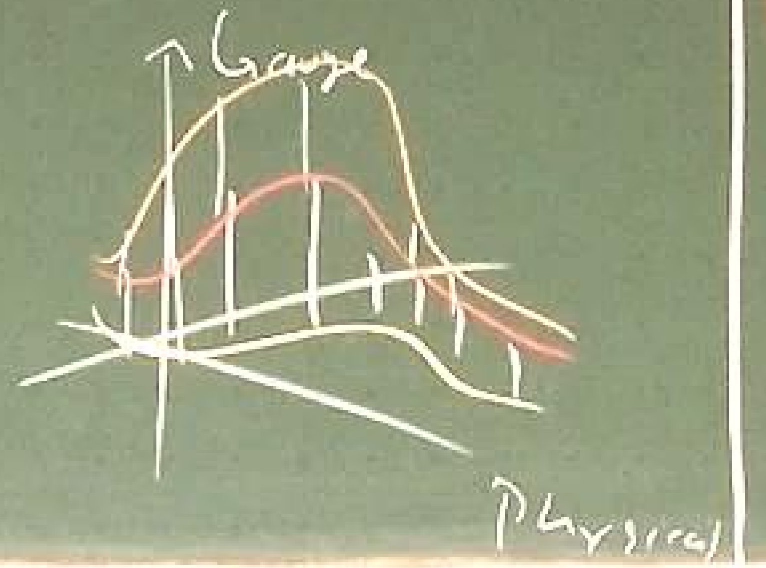
$$\mathcal{L} = -\frac{1}{4}F^2 + \left[(\partial_\mu + ieA_\mu)(v+h)e^{i\varphi}\right]\left[\ldots\right]^* + \mu^2 (v+h)^2 - \lambda (v+h)^4$$

$$= \underbrace{-\frac{1}{4}F^2 + e^2v^2A_\mu^2}_{\text{massive gauge field}} + \underbrace{(\partial_\mu h)^2 - m_h^2 h^2}_{\text{massive Higgs field}} \quad 4\lambda v^2$$

$$+ \underbrace{v^2(\partial_\mu\varphi)^2}_{\text{Massless Goldstone mode}} + \underbrace{2ev^2(\partial_\mu\varphi)A^\mu}_{\text{Quadratic coupling}} + \underbrace{\ldots\ldots}_{\text{Interactions}}$$

→ Lagrangian is still gauge invariant.

- $\tilde\varphi = \varphi + \alpha$
- $\tilde h = h$
- $\tilde A_\mu = A_\mu - \frac{1}{e}\partial_\mu \alpha$



6] Gauge fixing.

Unitary gauge. $\phi \doteq \phi^* \Leftrightarrow \varphi = 0$

$\tilde\phi = \phi_0\, e^{i\varphi} e^{-i\varphi}$

Gauge transformation. $\alpha(x) = -\varphi(x)$

$$\tilde\phi = e^{-i\varphi}\phi, \quad \tilde A_\mu = A_\mu + \frac{1}{e}\partial_\mu\varphi$$

$$\Downarrow$$

$$\boxed{\tilde{\mathcal{L}} = -\frac{1}{4}\tilde F^2 + e^2v^2\tilde A_\mu^2 + (\partial_\mu \tilde h)^2 - m_h^2 \tilde h^2 + \text{Interactions}}$$

→ Goldstone mode φ disappeared!

(φ is pure gauge DOF → not physical)

7| Consistency check:

$\#(\text{dof})$ before SSB $= 2$ (massless vector boson) $+ 2$ (complex scalar)

— " — after — " — $= 3$ (massive vector bosons) $+ 1$ (real scalar Higgs field)

10.2 The Standard Model

10.2.1. Preliminaries

1| Chiral projectors.

$$P_{\substack{R\\L}} = \frac{1}{2}\left(\mathbb{1} \pm \gamma^5\right) \overset{\text{Weyl}}{=} \begin{pmatrix} \overset{\mathbb{1}_2}{0} & 0 \\ 0 & \mathbb{1}_2 \end{pmatrix}$$

Chiral fermion fields:

$$\psi_R := P_R \psi$$

$$\psi_L := P_L \psi$$

2|

- $\overline{\psi_L} = \overline{\psi} P_R$ $\qquad \overline{\psi} = \psi^\dagger \gamma^0$

$\overline{\psi_R} = \overline{\psi} P_L$ $\qquad \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$

$$\psi = \underbrace{(P_R + P_L)}_{\mathbb{1}} \psi = \psi_R + \psi_L$$

$$\Downarrow$$

$$\boxed{\begin{aligned} \overline{\psi}(i\not{\partial} - m)\psi \overset{\circ}{=}\ & \overline{\psi_R}(i\not{\partial})\psi_R + \overline{\psi_L}(i\not{\partial})\psi_L \\ & - m\,\overline{\psi_L}\psi_R - m\,\overline{\psi_R}\psi_L \end{aligned}}$$

3| $[P_{R,L}, \Lambda_{\frac{1}{2}}] \overset{\circ}{=} 0$

$SO^+(1,3)$

$$e^{i[illegible]_{\alpha\mu} S^{\alpha\mu}} \longrightarrow S^{\alpha\mu} = \frac{i}{4}[\gamma^\alpha, \gamma^\mu]$$

Parity:

$$P = \gamma^0 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$$

→ Under additional (gauge) symmetry left- and right handed fields $\psi_{L,R}$ can transform under <u>different</u> representations

10.2.2 Overview →

• Scalar bosons (= Spin-0)

$\left.\text{2× Complex Higgs fields} \quad \phi^+, \phi^0\right\}$ Before Higgs SSB

$3\times \downarrow$ SSB

After Higgs SSB $\left\{ \text{1× Real Higgs field } h \right.$

Field content:

1) Fermions (= Spin-$\frac{1}{2}$)

Generation n	I	II	III
Leptons	$e_L \; e_R$ $\nu_{eL} \; (\nu_{eR})$	$\mu_L \; \mu_R$ $\nu_{\mu L} \; (\nu_{\mu R})$	$\tau_L \; \tau_R$ $\nu_{\tau L} \; (\nu_{\tau R})$
Quarks	$u_L \; u_R$ $d_L \; d_R$	$c_L \; c_R$ $s_L \; s_R$	$t_L \; t_R$ $b_L \; b_R$

$e^{ie\alpha(x)} \quad \psi \qquad e^{i\alpha^a t^a}$

Vector Bosons (= Spin-1)

Force	Electroweak	Strong
Gauge group	$SU(2)_L \times U(1)_Y$	$SU(3)_C$
# Generators	$3 + 1 = 4$	8
Gauge fields	$W_\mu^i \; (i = 1,2,3)$ B_μ before SSB	$G_\mu^a \; (a = 1, \dots, 8)$
Gauge bosons	γ, W^+, W^-, Z after Higgs SSB	8 Gluons

2) Lagrangian:

$$\boxed{\mathcal{L}_{SM} = \mathcal{L}_{EWS} + \mathcal{L}_{QCD} \left(+ \mathcal{L}_{GF} + \mathcal{L}_{Ghost} \right)}$$

(Standard Model)

Gauge fixing: $-\frac{(\partial^\mu A_\mu)^2}{2\xi}$

$\det\left(\frac{\delta G(A^\alpha)}{\delta \alpha}\right)$

- $\mathcal{L}_{EWS}$: Electroweak Standard Model
  = Glashow-Weinberg-Salam (GWS) Theory
  = Unification of weak & electromagnetic forces

- $\mathcal{L}_{QCD}$: Quantum Chromodynamics
  = Strong force