10. Excursions
10.1 The Higgs Mechanism

$\mathcal{L} = \frac{-1}{4} F^2 + |D\phi|^2 - V(\phi)$

Potential: $V(\phi) = m^2 |\phi|^2 + \lambda |\phi|^4$

Gauge symmetry: $U(1)$

$\bar{\theta} = e^{i \alpha} \theta$

$A_\mu = A_\mu^0 - \frac{2}{\epsilon} \theta^0 \phi^0$

$M^2 > 0$

$M^2 < 0$

Unique vacuum: $\langle \phi \rangle = \langle \bar{\phi} \rangle = 0$

Degenerate vacua: $\langle \phi \rangle = 1, 0 \pm \sqrt{\frac{2 F}{\lambda}}$

Vacuum expectation value (VEV)

Goldstone modes

Goldstone Theorem

Spontaneous breaking of global, continuous symmetry

Massless scalar particle in spectrum (Massive Goldstone Boson)

Examples:
- Pions ($\pi^0$, $\pi^+\pi^-$, $\pi^0\pi^0$)
- Magnons ($\sigma$, $\gamma$)

Exception:
- Superconductivity ($\text{SU}(2)_{\text{G}}$) but not Goldstone mode

How can Goldstone Theorem fail?
5) \[ \mathbf{X} < \phi > = \phi_c \] breaks global U(1) symmetry.

- \[ \mathbf{d} (\phi) = (\mathbf{v} + i \mathbf{k} \mathbf{w}) e^{i \mathbf{p} / m} \]

2 real fields:
- \[ \mathbf{v} \] Higgs field
- \[ \mathbf{k} \] Goldstone boson

\[ L = \frac{-1}{4} F^2 + \left[ (\partial \phi + i A_\mu \phi) (\partial \phi - i A_\mu \phi) \right] - \frac{\lambda}{2} (\phi^2)^2 \]

\[ A_\mu = A_\mu + \frac{1}{\mathbf{e}} \mathbf{A} \mathbf{e} \phi \]

6) Gauge theory:
- Unitary group: \[ \phi = \phi^* \iff \phi = 0 \]
- \[ \mathcal{G} = e^{- i A_\mu \phi} \mathcal{A}_\mu = A_\mu + \frac{1}{\mathbf{e}} \mathbf{A} \mathbf{e} \phi \]

Goldst
come mode + \[ \phi \] disappeared.

(\[ \phi \] is pure gauge D = not physical)
The Standard Model

10.2.1. Preliminaries

1. Chiral projectors:
   \[ \rho = \frac{1}{2} \left( \mathbb{1} + \gamma_5 \right) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]
   Chiral fermion fields:
   \[ \psi_R = \rho \psi \]
   \[ \psi_L = \rho \psi \]

2. [\[ P_R = \frac{1}{2} \left( \mathbb{1} + \gamma_5 \right) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \]
   \[ P_L = \frac{1}{2} \left( \mathbb{1} + \gamma_5 \right) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]
   \[ \psi_R = P_R \psi \]
   \[ \psi_L = P_L \psi \]
Under additional (gauge) symmetry, left- and right-handed fields \( \psi \) can transform under different representations.

10.2.2 Overview

- Scalar bosons \( (= Spin-0) \)
- 2x Complex Higgs fields \( \phi, \phi^\dagger \) \( (n=1) \)
- 3x Real Higgs field \( h \)

<table>
<thead>
<tr>
<th>Field content</th>
<th>11 Fermions (= Spin-1/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generation U</td>
<td>I</td>
</tr>
<tr>
<td>Leptons</td>
<td>e_L, e_R</td>
</tr>
<tr>
<td>Quarks</td>
<td>u_L, u_R</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Field content</th>
<th>11 Vector Bosons (Spin-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force</td>
<td>Electroweak</td>
</tr>
<tr>
<td>Gauge group</td>
<td>SU(2)_L X U(1)_Y</td>
</tr>
<tr>
<td># Generations</td>
<td>3 + 1 = 4</td>
</tr>
<tr>
<td>Gauge field</td>
<td>W^± (1, 2, 1)</td>
</tr>
<tr>
<td>Gauge bosons</td>
<td>( B_A ) before SSB</td>
</tr>
<tr>
<td>( \bar{W} )</td>
<td>( \frac{1}{2} ) W^\dagger W</td>
</tr>
<tr>
<td>( \bar{H_\tau} )</td>
<td>( \bar{H_\tau} ) SSB</td>
</tr>
</tbody>
</table>
Lagrangian:

\[ \mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{EWS}} + \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{GUT}} \]

(Standard Model)

\[ \mathcal{L}_{\text{EWS}} = -\frac{g^2}{2} \Omega \cdot E \cdot \mathcal{A} \cdot \text{det}(\mathcal{S}) \]

(Electroweak Standard Model)

\[ \mathcal{L}_{\text{GUT}} \]

Glashow-Weinberg-Salam (GWS) Theory

\[ \mathcal{L}_{\text{QCD}} \]

Unification of weak & electromagnetic forces

\[ \mathcal{L}_{\text{QCD}} \]

Quantum Chromodynamics

\[ \mathcal{L}_{\text{QCD}} = \text{Strong Force} \]