Organization

- $\boxed{\text{itp3.info/qft}}$
- Tutorials. 1 problemset/week
  written (80%) + oral (66%) + blackboard
  Friday 2nd block → next week

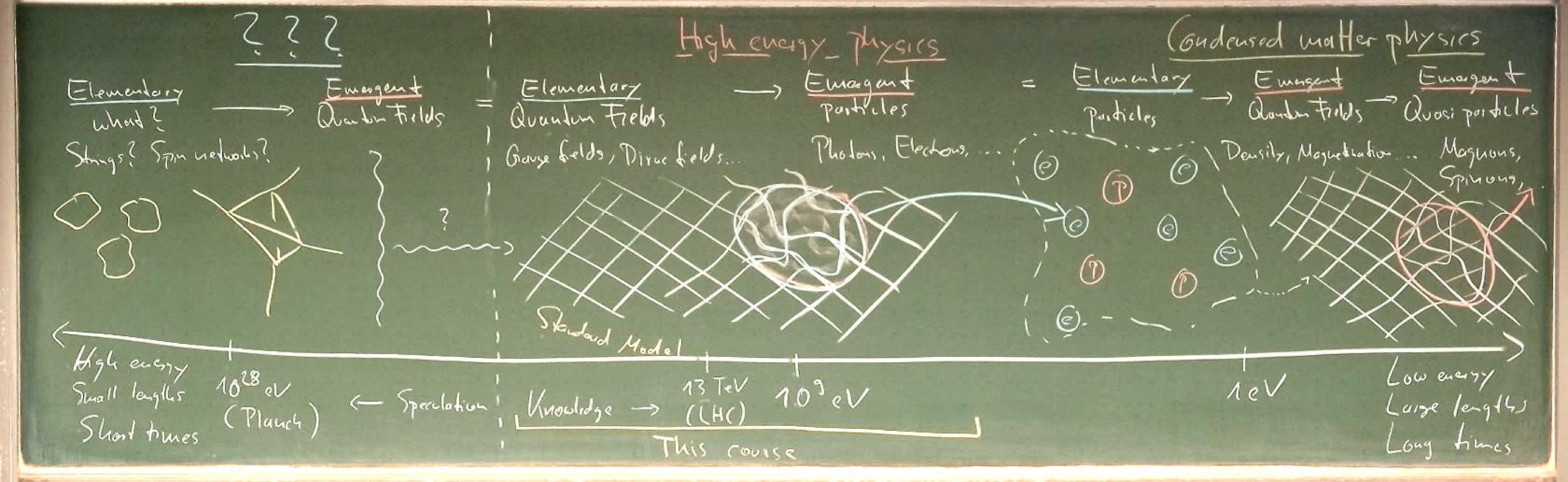
$$\vec{\mu} = g \frac{\mu_B}{\hbar} \vec{S}$$

Sign up: cms.itp3.info/tutorials/login

Lecture Key: $\boxed{\text{qftSS23!}}$

- Lecture: 2 per week (wed + fri)
  - MSc Physik
    - ↙ Ergänzung (9 ETCS)
    - ↘ Schwerpunkt (12 ETCS)
      ↓
      Vertiefung:
      5 additional lectures on Standard Model (24.07 - 28.07)
  - MSc Physics
    ↓
    Semicompulsory (9 ECTS)
- Oral exam (lecture free period)

- Based on Peskin & Schroeder
- Recordings → ILIAS
  Script
- Prerequisites:
  - QM (second quantization)
  - SRT (tensor calculus)
  - Complex analysis (residue thm)



? ? ?
High energy physics
Condensed matter physics
Elementary
what?
Emergent
Quantum Fields
=
Elementary
Quantum Fields
Emergent
particles
=
Elementary
particles
Emergent
Quantum Fields
Emergent
Quasi particles
Strings? Spin networks?
Gauge fields, Dirac fields...
Photons, Electrons, ...
Density, Magnetization ...
Magnons, Spinons, ...
?
Standard Model
High energy
Small lengths
Short times
10^{28} eV
(Planck)
← Speculation
Knowledge →
13 TeV
(LHC)
10^9 eV
1 eV
Low energy
Large lengths
Long times
This course

1. Elements of Classical Field Theory

1.1. Lagrangian and Hamiltonian Formalism

Recap: Classical mechanics of "points"

1| # DOF q_i label $i = 1, \ldots, N$

2| Lagrangian $L(\{q_i\}, \{\dot{q}_i\}, t) = T - V$

3| Action $S[q] = \int dt\, L(q(t), \dot{q}(t), t) \in \mathbb{R}$

$q(t)$

4| Hamilton's principle of least action:

$$\frac{\delta S[q]}{\delta q} \overset{!}{=} 0 \Leftrightarrow \delta S = \int dt\, \delta L \overset{!}{=} 0$$

5| Euler Lagrange equations. $i = 1, \ldots, N$

$$\boxed{\frac{\partial L}{\partial q_i} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} = 0}$$

Analogous: Lagrangian Field Theories

1] $\nexists$ One or more fields $\phi(x)$ on spacetime $x \in \mathbb{R}^{1,3}$, $\mathbb{R}^4$

with derivatives $\partial_\mu \phi(x)$

Γ $\partial_0 = \partial_t$, $\partial_{i=1,2,3} = \partial_{x,y,z}$

2] Lagrangian density:

$$\mathcal{L}(\phi, \partial\phi, \cancel{x})$$

$\rightarrow$ Lagrangian: $L = \int d^3x\, \mathcal{L}(\phi, \partial\phi, \vec{x})$

3] Action:

$$S[\phi] = \int dt\, L = \int dt \int d^3x\, \mathcal{L}(\phi, \partial\phi)$$

$$= \int d^4x\, \mathcal{L}(\phi, \partial\phi)$$

4] Action principle

$$0 \overset{!}{=} \delta S[\phi] = \int d^4x\, \delta \mathcal{L}(\phi, \partial_\mu\phi)$$

$$= \int d^4x \left\{ \frac{\partial \mathcal{L}}{\partial \phi} \delta\phi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \overbrace{\delta(\partial_\mu \phi)}^{\partial_\mu(\delta\phi)} \right\}$$

$$= \int d^4x \left\{ \frac{\partial \mathcal{L}}{\partial \phi} \delta\phi - \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right) \delta\phi + \overbrace{\partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta\phi\right)}^{F^\mu} \right\}$$

$$\overset{\text{Gauss}}{=} \int_\partial d\sigma_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \underbrace{\delta\phi}_{=0} + \underbrace{\int d^4x \left\{ \frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right\} \delta\phi}_{\overset{!}{=} 0 \ \forall \delta\phi}$$

5] Euler Lagrange Equation

$$\boxed{\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right) = 0}$$

Recap: Hamiltonian mechanics

Lagrangian $L(q,\dot{q},t) \xrightarrow[\text{Conjugate momenta}]{\text{Legendre Trafo.}}$ Hamiltonian $\boxed{H(q,p,t) = p\cdot\dot{q} - L(q,\dot{q},t)}$

$$p = \frac{\partial L}{\partial \dot{q}} \Leftrightarrow \dot{q} = \dot{q}(p)$$

Analogous: Hamiltonian Field Theory

1| $x = x_i \hat{=} i$ discrete spatial coordinates

$$\frac{\partial L}{\partial \dot{q}_i} = p_i \hat{=} p(x) = \frac{\partial L}{\partial \dot{\phi}(x)} = \frac{\partial}{\partial \dot{\phi}(x)} \int d^3y\, \mathcal{L}(\phi(y), \dot{\phi}(y))$$

$$\sim \sum_y{}' d^3y \underbrace{\frac{\partial}{\partial \dot{\phi}(x)} \mathcal{L}(\phi(y), \dot{\phi}(y))}_{\delta_{x,y} \left.\frac{\partial \mathcal{L}}{\partial \dot{\phi}}\right|_{y=x}} = \frac{\partial \mathcal{L}}{\partial \dot{\phi}(x)} d^3x$$

$\rightarrow$ Momentum density conjugate to ϕ is $\boxed{\Pi(x) = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}}$

2| Hamiltonian:

$$H = \sum_x{}' \underbrace{p(x)}_{\Pi(x)\, d^3x} \cdot \dot{\phi}(x) - \underbrace{L}_{\sum_x' \mathcal{L}(\phi(x), \dot{\phi}(x))\, d^3x} \longrightarrow \boxed{H = \int d^3x \{ \underbrace{\Pi(x)\dot{\phi}(x) - \mathcal{L}(\phi, \dot{\phi})}_{\text{Hamiltonian density } \mathcal{H}} \}}$$

Example 1.1 Free scalar field

1) Real field $\phi : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$

with $(\vec{x}, t) \to \phi(\vec{x}, t) = \phi(x)$

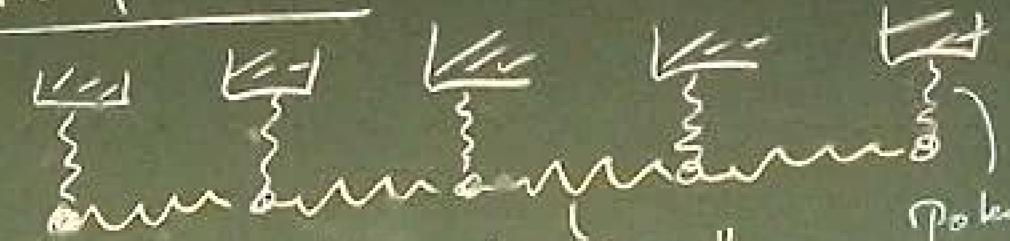
2) Lagrangian density:

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial_t \phi)^2 - \frac{1}{2}(\nabla\phi)^2} - \frac{1}{2} m^2 \phi^2$$

$$= \frac{1}{2}(\partial\phi)^2 - \frac{1}{2} m^2 \phi^2$$

$$(\partial\phi)^2 = (\partial_\mu \phi)^2 = (\partial_\mu \phi)(\partial^\mu \phi)$$

3) Interpretation



$\phi(x)$ kinetic energy/2 $(\partial_t\phi)^2$ "stretching" $(\nabla\phi)^2$ potential energy ϕ^2

4) EOM:

$$-m^2\phi - \partial_\mu(\partial^\mu \phi) = 0 \qquad \partial^2 = \Box$$

$$\Rightarrow \boxed{(\partial_\mu \partial^\mu + m^2)\,\phi = 0}$$

(classical) Klein-Gordon equation

5) Conjugate momentum field:

$$\pi = \frac{\partial \mathcal{L}}{\partial \dot\phi} = \dot\phi$$

6) Hamiltonian (density):

$$\mathcal{H} = \pi\dot\phi - \frac{1}{2}\dot\phi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2} m^2 \phi^2$$

$$= \frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2} m^2 \phi^2$$

1.2. Symmetries and Conservation Laws

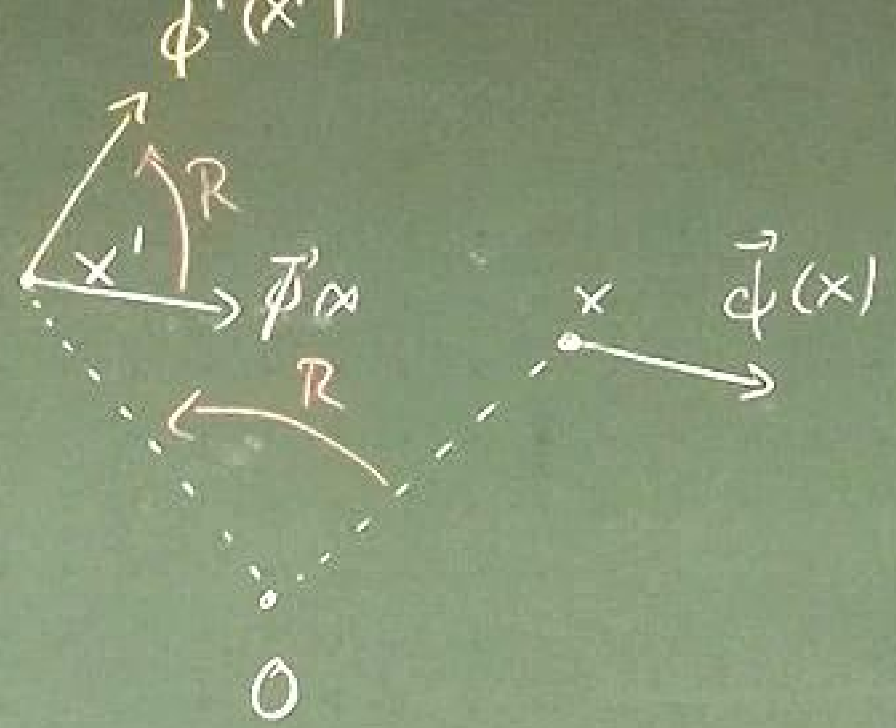
1] ∡ Transformations of coordinates and fields.

$$x \longmapsto x' = x'(x) \quad \text{and} \quad \phi(x) \longmapsto \phi'(x') = \mathcal{F}(\phi(x))$$

Two effects: coordinates and fields transform

Example 1.2: Rotation of a vector field $\vec{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$

and $R \in SO(3)$



$$\Rightarrow \quad x' = Rx$$

$$\phi'(x') = R\phi(x)$$

$$= \underbrace{R}_{\text{vector field}}\phi(R^{-1}x')$$