Recap: $\boxed{\mathcal{L}_{SM} = \mathcal{L}_{EWS} + \mathcal{L}_{QCD}}$

10.23. The Glashow Weinberg Salam Theory

$$\boxed{\mathcal{L}_{EWS} = \mathcal{L}_{\text{Fermion}} + \mathcal{L}_{\text{Yang-Mills}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}}$$

- $\mathcal{L}_{\text{Fermion}}$: Kinetic energy of fermions + Electroweak interaction (e.g. β-decay)
- $\mathcal{L}_{\text{Yang-Mills}}$: Kinetic energy of gauge bosons + Interactions between gauge bosons
- $\mathcal{L}_{\text{Higgs}}$: SSB of Higgs field (+Masses of $W^\pm$, Z)
- $\mathcal{L}_{\text{Yukawa}}$: today

Remember:

- $\Psi_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \ldots$ Isospin doublets
- $\Psi_R = u_R, d_R, e_R, \ldots$ Isospin singlets
- $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ Isospin doublet

$\downarrow$ Higgs mechanism

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

v: VEV; $h(x)$: real, scalar Higgs field

- **Electric charge:**

$$\boxed{SU(2)_L \times U(1)_Y \xrightarrow{\text{SSB}} U(1)_Q}$$

with $\boxed{Q = T^3 + Y} \in su(2)_L \oplus u(1)_Y$

$\Downarrow$

fix Hyperchange.

$$Y(e_L) = Q(e_L) - T^3(e_L) = -1 - \left(\tfrac{1}{2}\right) = -\tfrac{1}{2}$$

$$Y(e_R) = Q(e_R) - T^3(e_R) = -1 - 0 = -1$$

$$Y(\phi) = \tfrac{1}{2}$$

chosen

10| Higgs mechanism Part II: Masses for the fermions

i) How to form gauge invariant terms include left- and right handed fermions?

$$\boxed{-y_e \cdot \overline{\Psi}_L \cdot \Phi \, e_R + h.c} \quad (*) \qquad \Psi_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$$

$$(\overline{\nu}_{eL} \; \overline{e}_L) \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix} e_R$$

• $SU(2)_L$ invariant

$$Y(\Phi) + Y(e_R) - Y(\Psi_L) = \frac{1}{2} - 1 - \left(-\frac{1}{2}\right) = 0 \Rightarrow U(1)_Y \text{ invariant}$$

• $(\overline{\Psi}_L \Phi) e_R = \underbrace{\phi^+}_{0} \overline{\nu}_{eL} e_R + \underbrace{\phi_0}_{\frac{1}{\sqrt{2}}(v + h(x))} \overline{e}_L e_R$

Higgs SSB

$$(*) = \boxed{-\underbrace{\boxed{y_e \frac{v}{\sqrt{2}}}}_{m_e = y_e \frac{v}{\sqrt{2}}} (\overline{e}_L e_R + \overline{e}_R e_L)}$$

Dirac mass

Yukawa interaction: $\overline{\Psi} \Psi \phi$

$\overline{\Psi} \gamma^\mu \Psi A_\mu$

ii|

$$\boxed{\mathcal{L}_{Yukawa} = -\Gamma^u_{mn} \overline{Q}^m_L \hat{\Phi} u^n_R - \Gamma^d_{mn} \overline{Q}^m_L \Phi d^n_R - \Gamma^l_{mn} \overline{L}^m_L \Phi l^n_R}$$

• $m, n \in$ I, II, III : fermion generations (summation implied)

• $x \in \{u, d, l, \nu\}$: fermion types

$l^{I}_R = e_R$, $l^{II}_R = \mu_R$

$u^{I}_R = u_R$, $u^{II}_R = c_R$

$$\mathcal{L}_{\text{Yukawa}} = -\Gamma^u_{mn}\,\bar{Q}^m_L\,\hat{\Phi}\,u^n_R - \Gamma^d_{mn}\,\bar{Q}^m_L\,\Phi\,d^n_R - \Gamma^l_{mn}\,\bar{L}^m_L\,\Phi\,l^n_R - \Gamma^\nu_{mn}\,\bar{L}^m_L\,\hat{\Phi}\,\nu^n_R + \text{h.c.}$$

Γ^X_{mn}: coupling constants ($\Gamma^l_{11} = \gamma_e$)

Q^m_L, L^m_L: left-handed quark resp. lepton doublets of gen. m

Γ $L^I_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$ $\bar{\nu}_{eL}\,\nu_{eR}$

- $\hat{\Phi}_i \equiv \varepsilon^{ij}\Phi^*_j$ Higgs doublet with opposite Hypercharge. $Y(\hat{\Phi}) = -\frac{1}{2}$

iii] • Yukawa term generates fermion masses (but not for neutrinos if right-handed neutrinos are missing)

→ Neutrino masses

- ———— leads to generation changing transitions of quarks (CKM matrix)
- ———— Neutrino oscillations (if ν_R exist) (PNMS matrix)

10.2.4. Quantum Chromodynamics

1] Gauge Symmetry

$$\boxed{SU(3)_c}$$

color charge

→ 8 generators K^a $\quad a = 1 \dots 8$

$$[K^a, K^b] = i f^{abc} K^c$$

→ Irreducible representations:

- 1D: Trivial (Singlet). $\hat{K}^a = 0$
- 3D: Defining (Triplet)

$\hat{\lambda}^a = \frac{\lambda_a}{2}$

with λ_a 3x3 Hermitian matrices
(Gell-Mann matrices)

2| Field representations:

- Quarks = $SU(3)_c$ triplets "colors"

$$q = \begin{pmatrix} q_r \\ q_g \\ q_b \end{pmatrix} \quad \text{for } q \in \{u, d, c, s, t, b\}$$

red
green
blue

- Leptons + Higgs = $SU(3)_c$ singlets

→ Gauge transformation of fields:

$$\boxed{\text{Quark triplet} \quad \tilde{q} = \underbrace{e^{i \hat{\lambda}^a \beta^a(x)}}_{U_c(x)} q}$$

3| Lagrangian

$$\boxed{\begin{gathered} \mathcal{L}_{QCD} = \sum_q \bar{q}(i \not{D}_c) q - \frac{1}{4} \left(G^a_{\mu\nu}\right)^2 \\ D_{c\mu} = \partial_\mu - i g_s G^a_\mu \hat{\lambda}^a \end{gathered}}$$

g_s: coupling constant of strong force

G^a_μ: 8 gauge fields → 8 gauge bosons
→ 8 Gluons

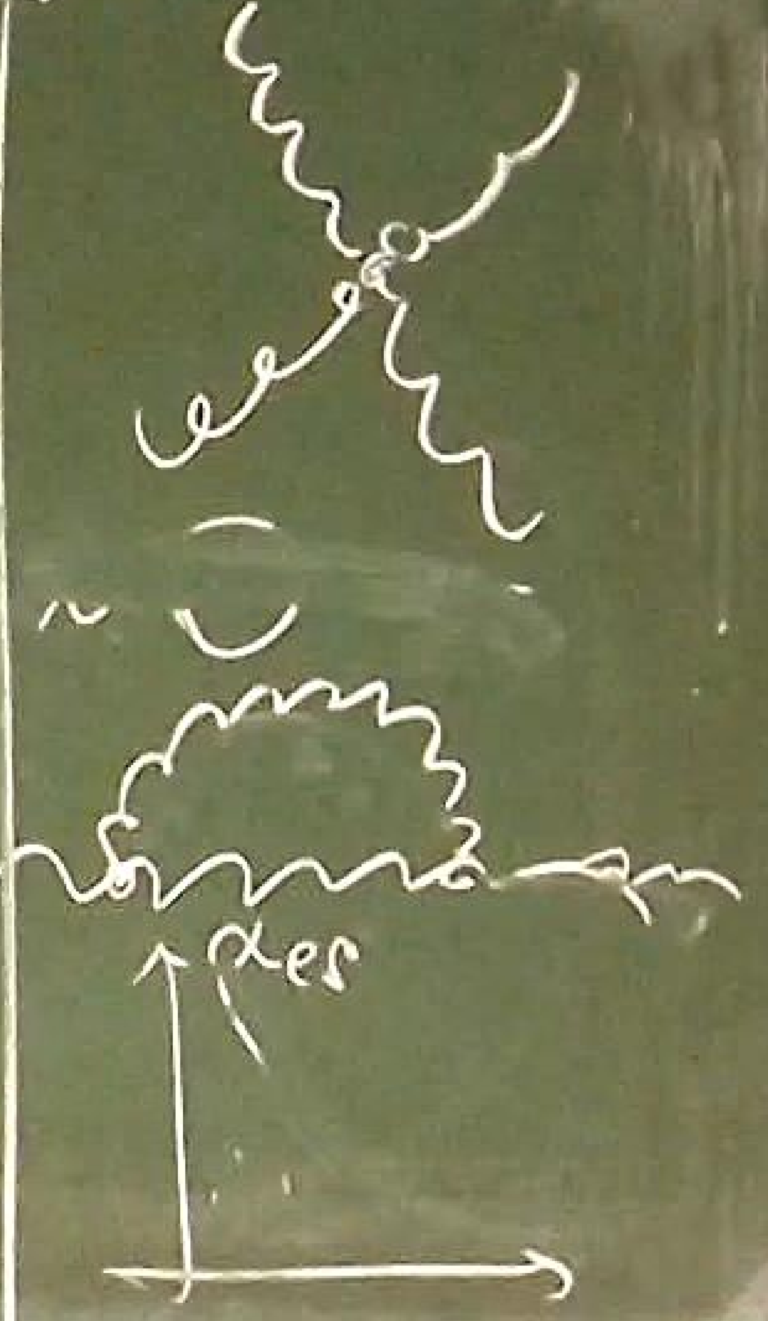
$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu$$

Renormalization:

$\alpha_s = \frac{g_s^2}{4\pi}$

$\alpha^{eff}(q^2) \xrightarrow{q^2 \to \infty} 0$ Asymptotic freedom

$\alpha^{eff}(q^2) \xrightarrow{q^2 \to 0} \infty$ (confinement)



10.5.7. Summary

$$\boxed{\begin{array}{c} \underbrace{SU(2)_L \times U(1)_Y}\times SU(3)_C \\ \Big\downarrow \text{ SSB, Higgs} \\ U(1)_Q \\ \text{(electric charge)} \end{array}}$$

Fields (include ν_R)

[2 Leptons + 2 Quarks × 3 Colors] × 3 Generations

= 24 $\underbrace{\text{Dirac bispinors}}_{\text{4 complex fields}}$ = 96 Complex fields

Parameters: (without ν_R)

× 9 × Fermion masses

× 1 m_h Higgs mass

× 1 Higgs field VEV v

× 3 Gauge field couplings g, g', g_s

× 4 × CKM matrix $\theta_{12}, \ldots$

18 parameters.