Recap:

10.2. The Standard Model

10.2.1. Preliminaries

Chiral projectors: $P_R = \frac{1}{2}(1+\gamma^5) = \begin{pmatrix} 0 & 0 \\ 0 & 1\!\!1_2 \end{pmatrix}$

Chiral fermion fields: $\psi_R = P_R \psi$

$\psi_L = P_L \psi$

Dirac Lagrangian:

$$\bar\psi(i\partial\!\!\!/ - m)\psi = \boxed{\bar\psi_L i\partial\!\!\!/ \psi_L + \bar\psi_R i\partial\!\!\!/ \psi_R - m\bar\psi_L\psi_R - m\bar\psi_R\psi_L}$$

all terms $SO^+(1,3)$ inv. mixes L and R

10.2.2. Overview

Fields:

Fermions:

	I	II	III	
	$e_L \quad e_R$	$\mu_L \quad \mu_R$	$\tau_L \quad \tau_R$	Leptons
Flavors	$\nu_{eL} \quad (\nu_{eR})$	$\nu_{\mu L} \quad (\nu_{\mu R})$	$\nu_{\tau L} \quad (\nu_{\tau R})$	
	$u_L \quad u_R$	$c_L \quad c_R$	$t_L \quad t_R$	Quarks
	$d_L \quad d_R$	$s_L \quad s_R$	$b_L \quad b_R$	

Generations

Vector bosons (Spin 1):

3 + 1 generators

Gauge fields

Electroweak force: $SU(2)_L \times U(1)_Y \to \begin{cases} W^i_\mu & (i=1,2,3) \\ B_\mu \end{cases}$

Strong force: $SU(3)_c \to G^a_\mu \quad (a=1,\dots,8)$

8 generators

Scalar bosons (Spin 0):

Higgs fields: $\phi^+, \phi^0 \in \mathbb{C}$

4 real dof.

Lagrangian:

$$\boxed{\mathcal{L}_{SM} = \mathcal{L}_{EWS} + \mathcal{L}_{QCD}}$$

ignoring ghosts!

Electroweak Unification (QED, Weak force, Higgs)

$\Downarrow$

Glashow-Weinberg-Salam theory

Quantum Chromodynamics (strong force)

0.2.3. The Glashow-Weinberg-Salam Theory

1] Lagrangian:

$$\boxed{\mathcal{L}_{EWS} = \mathcal{L}_{Fermion} + \mathcal{L}_{Yang\text{-}Mills} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}}$$

2] Gauge symmetry:

$$\underset{\text{isospin}}{\text{Weak}} \rightarrow \boxed{SU(2)_L \times U(1)_Y} \leftarrow \underset{\text{hypercharge}}{\text{Weak}}$$

Lie A.: $[T^i, T^j] = i\epsilon^{ijk} T^k$, $[Y, T^i] = 0$

Irreps:
- $\hat{T}^i = 0$ (1D: Singlet)
- $\hat{T}^i = \frac{\sigma^i}{2}$ (2D: Doublet)

$\hat{Y} = Y \cdot \mathbb{1}$

Hypercharge ($\mathbb{R}$)

EV of $\hat{T}^3$: Weak isospin ($\frac{1}{2}\mathbb{N}$)

3] $SU(2)_L$ Representations:

- Left-handed field = Isospin doublets

$$\Psi_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$$

→ Weak isospin: $T^3(\nu_{eL}) = +\frac{1}{2}$

$T^3(e_L) = -\frac{1}{2}$

$$\nu_{eL}(x) = \psi_L(x) \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\in L^2(\mathbb{R}^{1,3}) \otimes \mathbb{C}^4 \otimes \mathbb{C}^2$$

$$T^3(\nu_{eL}) = +\tfrac{1}{2} \Leftrightarrow \hat{T}^3 \nu_{eL}(x) = \psi_L(x)\, \frac{\sigma^3}{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = +\tfrac{1}{2}\, \nu_{eL}(x)$$

- Right-handed field = Isospin singlet

$$\psi_R = u_R, d_R, e_R, c_R, s_R, \mu_R, t_R, b_R, \tau_R$$

→ $T^3(e_R) = 0$

- Higgs field = Isospin doublet.

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \rightarrow T^3(\phi^+) = +\tfrac{1}{2}$$

$T^3(\phi^0) = -\frac{1}{2}$

Gauge transformations on fields:

LH doublets: $\tilde{\Psi}_L = e^{i\hat{Y}_L\alpha(x)} \underbrace{e^{i\hat{T}^i\beta^i(x)}}_{\to V_L} \Psi_L$

RH singlets: $\tilde{\psi}_R = e^{i\hat{Y}_R\alpha(x)} \psi_R$

Higgs doublet: $\tilde{\Phi} = e^{i\hat{Y}_H\alpha(x)} e^{i\hat{T}^i\beta^i(x)} \Phi$

$\hat{Y}_H = Y \cdot \mathbb{1}$

not yet fixed

4] Kinetic energy for fermions + Minimal coupling:

$$\boxed{\mathcal{L}_{\text{Fermion}} = \sum_{\Psi_L} \bar{\Psi}_L (i\not{D}_L) \Psi_L + \sum_{\psi_R} \bar{\psi}_R (i\not{D}_R) \psi_R}$$

- $D_{L\mu} = \partial_\mu - ig W_\mu^i \hat{T}^i - ig' B_\mu \hat{Y}_L$

coupling constants

$D_{R\mu} = \partial_\mu \quad 0 \quad - ig' B_\mu \hat{Y}_R$

singlet

$\longrightarrow$ Transformation of gauge fields:

$\tilde{B}_\mu = B_\mu + \frac{1}{g'} \partial_\mu \alpha, \quad \tilde{W}_\mu = V_L \left[W_\mu + \frac{i}{g} \partial_\mu \right] V_L^\dagger$, $\quad W_\mu = W_\mu^i \hat{T}^i$

Example:

$= (\bar{u}_L \bar{d}_L)(i\not{D}_L)\begin{pmatrix} u_L \\ d_L \end{pmatrix} + (\bar{\nu}_{eL} \bar{e}_L)(i\not{D}_L)\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} + \ldots$ ⊖

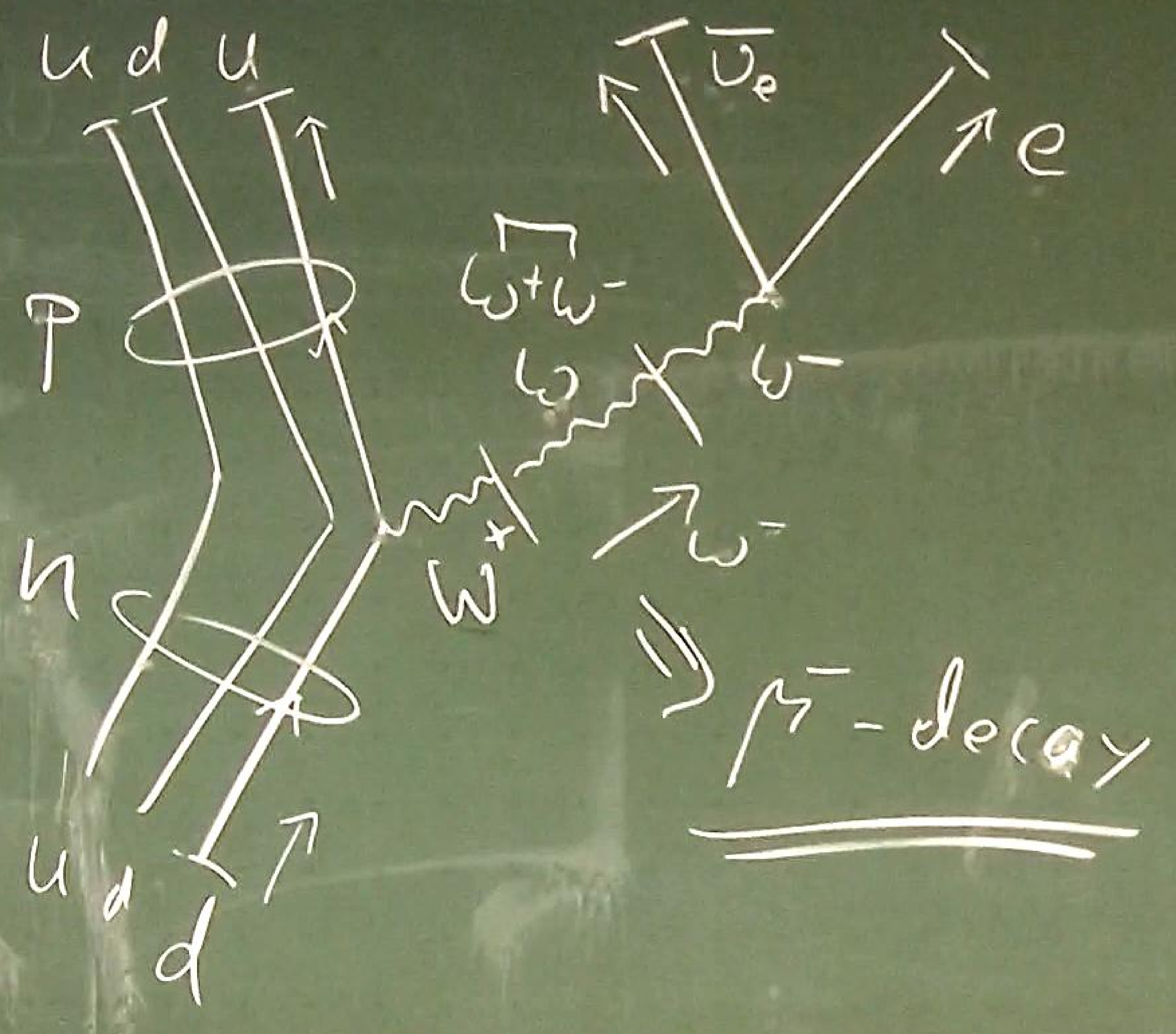
$D_{L\mu} = -ig(W_\mu^1 \hat{T}^1 + W_\mu^2 \hat{T}^2) + \ldots$

$= -i\frac{g}{\sqrt{2}} \begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix} + \ldots$

$W_\mu^\pm := \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2)$

⊖ $W_\mu^+ \bar{u}_L \gamma^\mu d_L + W_\mu^- \bar{e}_L \gamma^\mu \nu_{eL} + h.c. + \ldots$

$\int \quad a \quad + \quad b^\dagger$



5| Dirac mass?

$m(\bar{\Psi}_L \psi_R + \bar{\Psi}_R \psi_L) \rightarrow$ Undefined!

→ Not $SU(2)_L$ gauge invariant

because χ_L is compound of doublet

but Y_R is $SU(2)$ singlet.

$Ep^2 \rightarrow$ not Lorentz invariant

→ Cannot add mass term for fermions!

6| Kinetic energy for gauge bosons:

→ Yang-Mills-Lagrangian

$$\boxed{\mathcal{L}_{\text{Yang-Mills}} = -\frac{1}{4}(B^{\mu\nu})^2 - \frac{1}{4}(W^i_{\mu\nu})^2}$$

· $B^{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

· $W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + \underbrace{g\,\varepsilon^{ijk} W^j_\mu W^k_\nu}_{\text{interaction between gauge bosons.}}$

] Higgs field:

$$\boxed{\mathcal{L}_{Higgs} = (D^\mu_H \Phi)^\dagger (D_{H\mu} \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2} \quad (*)$$

$$D_{H\mu} = \partial_\mu - i g W^i_\mu \hat{T}^i - i g' B_\mu \hat{Y}_H$$

$$(\Phi^\dagger \Phi)^2 \neq |\phi^+|^4 + |\phi^0|^4$$

8] Higgs mechanism (Part 1).

i] $\mu^2 < 0 \rightarrow$ Non-zero VEV of Higgs field

W.l.o.g. $\langle \Phi \rangle = \Phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$ with $v = \sqrt{\frac{-\mu^2}{\lambda}}$

ii] Define electric charge operator.

$$\boxed{Q = T^3 + Y \quad \in \quad su(2)_L \oplus u(1)_Y}$$

$\rightarrow$ Choose: $Y(\Phi) = +\frac{1}{2}$

$$\hat{Q} \Phi_0 = \underbrace{\hat{T}^3 \Phi_0}_{-\frac{1}{2}\Phi_0} + \underbrace{\hat{Y} \Phi_0}_{\frac{1}{2}\Phi_0} = \left(-\tfrac{1}{2} + \tfrac{1}{2}\right) \Phi_0 = 0$$

$\Rightarrow e^{i \hat{Q} \alpha(x)} \Phi_0 = \Phi_0$

$\rightarrow$ Gauge symmetry $U(1)_Q$ is unbroken:

$$\boxed{SU(2)_L \times U(1)_Y \xrightarrow{3 \times SSB} \underbrace{U(1)_Q}_{\text{Unbroken gauge symmetry of QED}}}$$

iii] Fluctuations of Φ around Φ_0 in the unitary gauge.

$$\underset{\uparrow \text{ unitary gauge}}{\Phi(x) =} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

$h(x)$: real scalar Higgs field

[V] $\Phi(x)$ in $(*)$

$$(D_H^\mu \Phi^\dagger)(D_{H\mu}\Phi) \hat{=} \frac{v^2}{8}\left\{ g^2\left[(W_\mu^1)^2+(W_\mu^2)^2\right]+\left(-gW_\mu^3+g'B_\mu\right)^2\right\}+\dots$$

$$= \underbrace{\left(\frac{gv}{2}\right)^2}_{m_W^2} W_\mu^+ W^{-\mu} + \frac{1}{2}\underbrace{\left(\frac{v}{2}\right)^2(g^2+g'^2)}_{m_Z^2} Z_\mu^2 + \dots$$

[ ] Define new fields

$$\boxed{\begin{aligned} W_\mu^\pm &:= \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right) \\ Z_\mu &:= \frac{1}{\sqrt{g^2+g'^2}}\left(gW_\mu^3 - g'B_\mu\right) \\ A_\mu &:= \frac{1}{\sqrt{g^2+g'^2}}\left(g'W_\mu^3 + gB_\mu\right) \end{aligned}}$$

- A_μ: massless neutral gauge field of QED
- $W_\mu^\pm$: massive charged gauge bosons
- Z_μ: massive neutral gauge bosons

} weak interaction

$$\boxed{D_{H\mu} = \partial_\mu - (\dots) - i\boxed{\frac{gg'}{\sqrt{g^2+g'^2}}} A_\mu Q}$$

e electric charge

[g]
- $Y(e_L) = Q(e_L) - T^3(e_L)$
  $= \boxed{-1} - \left(-\frac{1}{2}\right) = \underline{\underline{-\frac{1}{2}}}$
  observation
- $Y(e_R) = Q(e_R) - T^3(e_R)$
  $= -1 - 0 = \underline{\underline{-1}}$