Recap:

0.1 The Higgs Mechanism

Complex scalar field ϕ + Maxwell:

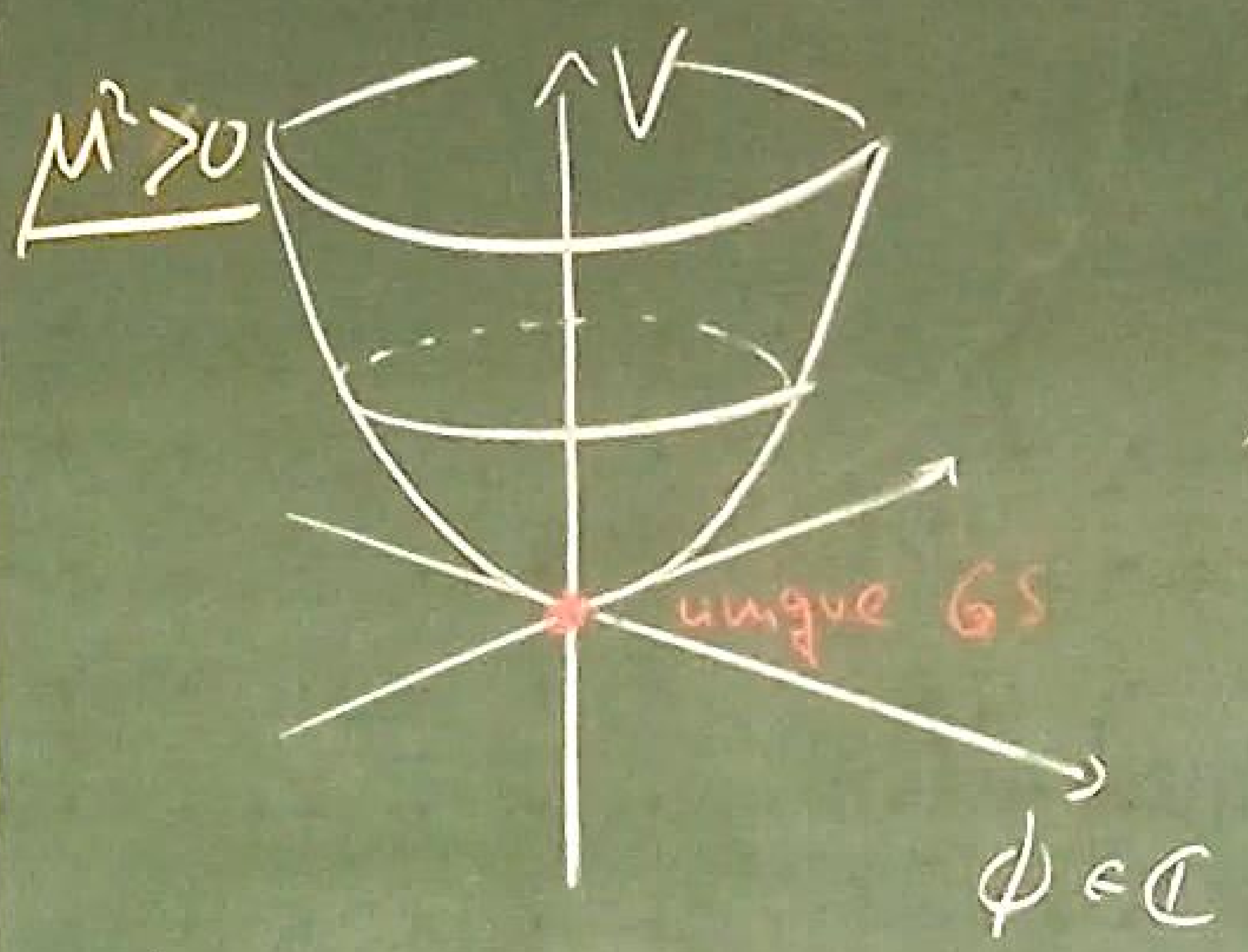
$$\boxed{\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + |D_\mu \phi|^2 - V(\phi)}$$

- $V(\phi) = \mu^2 |\phi|^2 + \lambda |\phi|^4$
- $D_\mu = \partial_\mu + ieA_\mu$

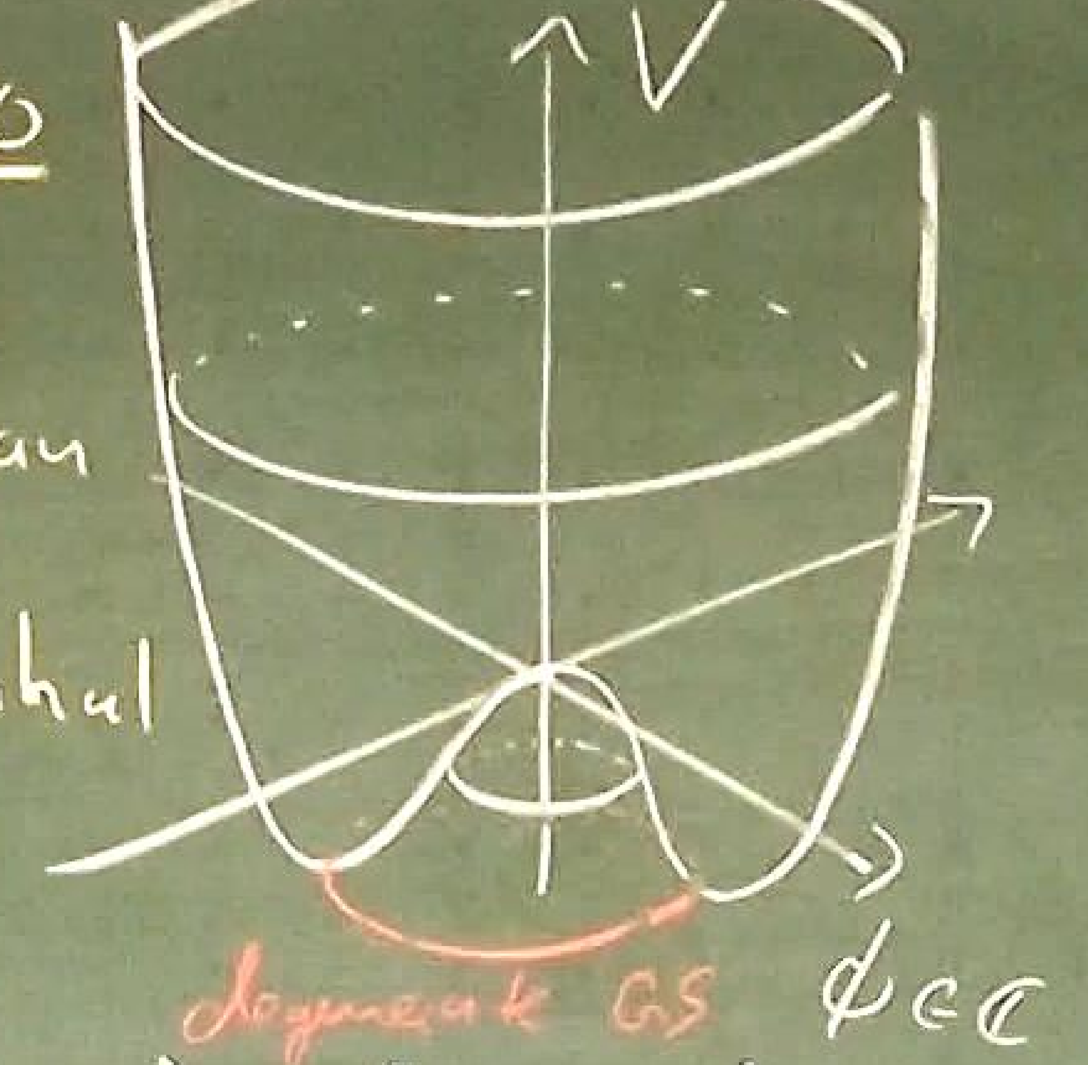
2) U(1) gauge transformations:

$$\tilde{\phi}(x) = e^{i\alpha(x)}\phi(x), \quad \hat{A}_\mu(x) = A_\mu(x) - \frac{1}{e}\partial_\mu \alpha(x)$$

3) Spontaneous U(1) Symmetry Breaking:



$\mu^2 < 0$
⇓
Mexican hat potential



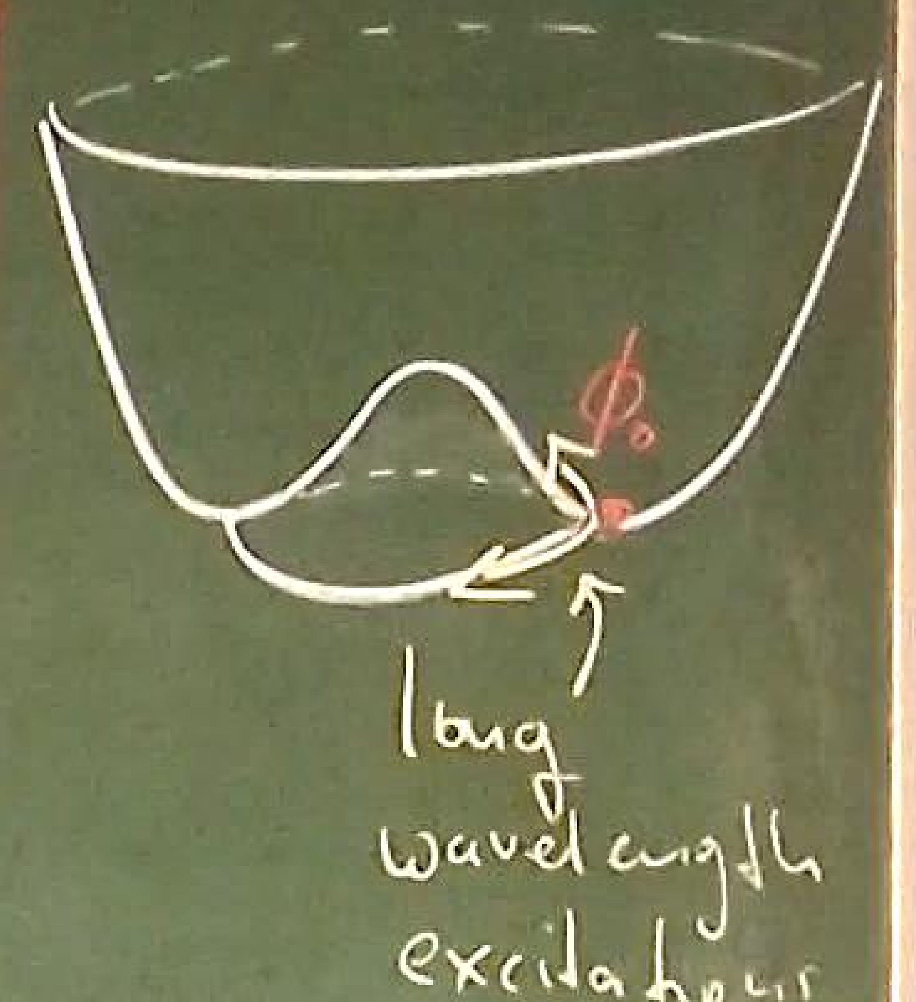
Vacuum expectation value (VEV) for $\mu^2 < 0$:

$$\phi_0 = \langle \phi \rangle \text{ with } v = |\phi_0| = \sqrt{\frac{-\mu^2}{2\lambda}} \neq 0$$

picks a random phase

4) Goldstone Theorem:

$$\boxed{\begin{array}{c}\text{Spontaneous breaking of global + continuous symmetry} \\ \Downarrow \\ \text{Massless Spin-0 excitation} \\ \text{(Nambu-) Goldstone Boson} \\ \text{(one for each broken generator)}\end{array}}$$

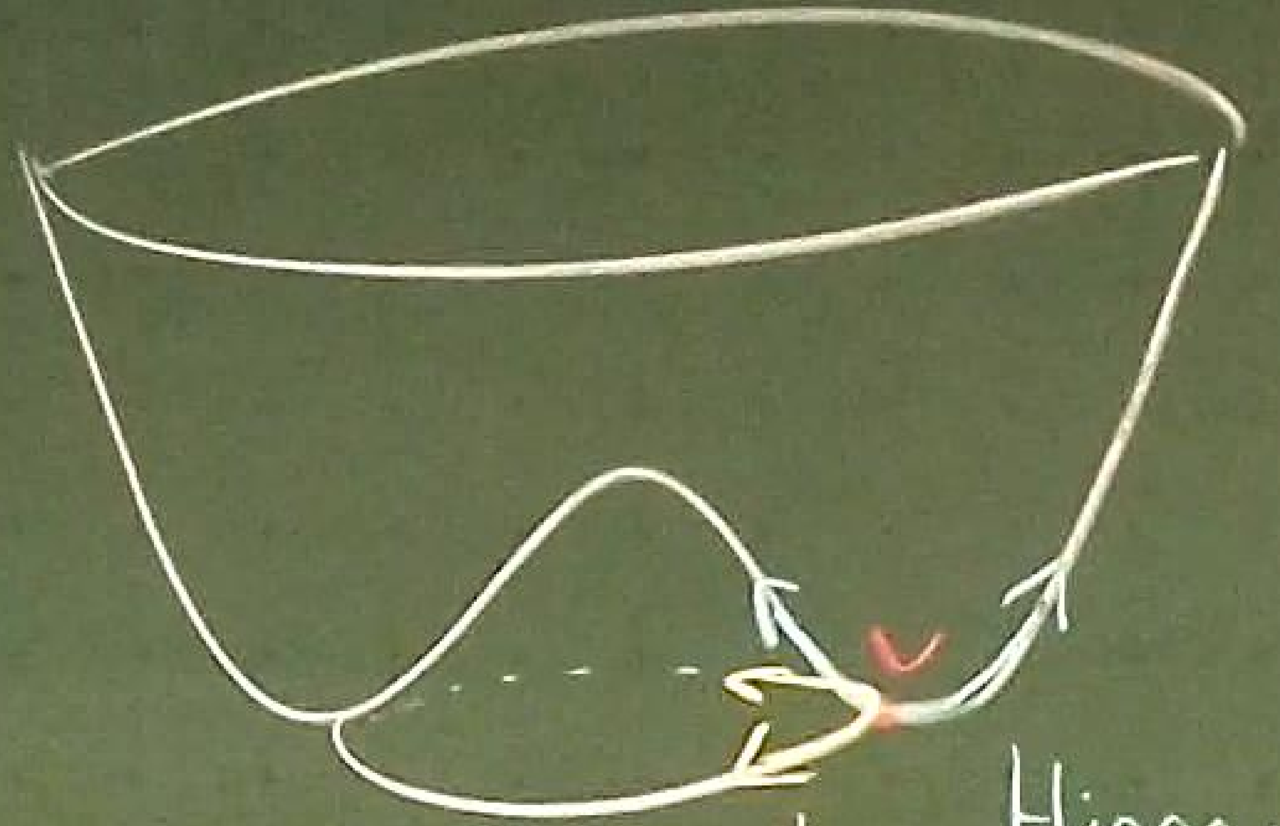


How can the Goldstone Theorem fail? Example: Superconductivity (broken U(1) symmetry, no Goldstone Mode)

5) $\phi_0 = \langle \phi \rangle = v$ breaks global U(1) symmetry.

$$\phi(x) = (v + \underline{h(x)})\, e^{i\varphi(x)}$$

$\downarrow$



$$\mathcal{L} = \underbrace{-\frac{1}{4}(F_{\mu\nu})^2 + e^2 v^2 A_\mu^2}_{\text{Massive gauge field}}$$

$$+ \underbrace{(\partial_\mu h)^2 - m_h^2 h^2}_{\text{Higgs field}} \qquad m_h^2 = 4\lambda v^2$$

$$+ \underbrace{v^2(\partial_\mu \varphi)^2}_{\text{Massless Goldstone mode}} + \underbrace{2ev^2(\partial_\mu\varphi)A^\mu}_{\text{Quadratic coupling}} + \text{Interactions}$$

Use: $-\mu^2 = 2\lambda v^2$

Note: $\mathcal{L}$ still gauge invariant:

- $\tilde{\varphi} = \varphi + \alpha$
- $\tilde{h} = h$
- $\tilde{A}_\mu = A_\mu - \frac{1}{e}\partial_\mu \alpha$

6) Fix gauge in unitary gauge:

$$\phi \overset{!}{=} \phi^* \iff \varphi \equiv 0$$

Gauge trafo. $\alpha(x) = -\varphi(x)$

$$\tilde{\phi} = \phi\, e^{i\alpha} \in \mathbb{R}$$

$$\to \boxed{\tilde{\mathcal{L}} = \underbrace{-\frac{1}{4}(\tilde{F}_{\mu\nu})^2 + e^2 v^2 \tilde{A}_\mu^2}_{\text{Massive gauge field}} + \underbrace{(\partial \tilde{h})^2 - m_h^2 \tilde{h}^2}_{\text{Massive Higgs field}} + \text{Interactions}}$$

$\to$ Goldstone mode φ disappeared!

Reason: φ is pure gauge d.o.f. and therefore unphysical!

7) Consistency check:

#(DOF) before SSB

= 2 (massless vector boson)

+ 2 (complex scalar field) = $\underline{\underline{4}}$

#(DOF) after SSB

= 3 (massive vector boson)

+ 1 (real scalar Higgs field) = $\underline{\underline{4}}$

10.2. The Standard Model

10.2.1. Preliminaries

1) Chiral projectors:

$$P_R = \frac{1}{2}(\mathbb{1}_4 + \gamma^5) \overset{\text{Weyl}}{=} \begin{pmatrix} \mathbb{1}_2 & \\ & 0 \end{pmatrix}$$

$$P_L = \frac{1}{2}(\mathbb{1}_4 - \gamma^5) \overset{\text{Weyl}}{=} \begin{pmatrix} 0 & \\ & \mathbb{1}_2 \end{pmatrix}$$

Chiral fermion fields:

$$\psi_R := P_R \psi, \quad \psi_L = P_L \psi$$

$$\psi = \underbrace{(P_R + P_L)}_{\mathbb{1}} \psi = \psi_R + \psi_L$$

2) Use $\overline{\psi} P_{R/L} = \overline{\psi_{L/R}}$ to show:

$$\left.\begin{aligned} \overline{\psi}(i\partial\!\!\!/ - m)\psi &\overset{!}{=} \overline{\psi_R} i\partial\!\!\!/ \psi_R + \overline{\psi_L}(i\partial\!\!\!/)\psi_L \\ &\quad - m\,\overline{\psi_L}\psi_R - m\,\overline{\psi_R}\psi_L \end{aligned}\right\} (*)$$

3) $[P_{R/L}, \Lambda_{\frac{1}{2}}] = 0$

$\sim [\gamma^\mu, \gamma^\nu]$

$SO^+(1,3)$

→ Terms in (*) are Lorentz invariant

10.2.2. Overview

1) Field content:

- **Fermions** (= Spin-$\frac{1}{2}$)

Generation	I	II	III
	e_L e_R	μ_L μ_R	τ_L τ_R
Leptons	ν_{eL} (ν_{eR})	$\nu_{\mu L}$ $(\nu_{\mu R})$	$\nu_{\tau L}$ $(\nu_{\tau R})$
Quarks	u_L u_R	c_L c_R	t_L t_R
	d_L d_R	s_L s_R	b_L b_R

- **Vector bosons** (= Spin 1)

Force	Electroweak	Strong
Gauge group	$SU(2)_L \times U(1)_Y$	$SU(3)_C$
# Generators	$3 + 1 = 4$	8
Gauge fields	W_μ^i $(i=1,2,3)$ B_μ Before SSB	G_μ^a $(a=1 \dots 8)$
Gauge bosons	γ, W^+, W^-, Z After SSB	8 Gluons

- **Scalar boson** (Spin 0)

Before Higgs SSB

$2\times$ Complex Higgs field: ϕ^+, ϕ^0

$\downarrow$ $3\times$ SSB

$1\times$ Real Higgs field h

After Higgs SSB

How to build a consistent QFT?

$$\mathcal{L}_{SM} = \mathcal{L}_{EWS} + \mathcal{L}_{QCD}$$

$$\left(+ \mathcal{L}_{GF} + \mathcal{L}_{Ghost} \right)$$

$$-\frac{(\partial_\mu A)^2}{2\xi} \qquad \det\left(\frac{\delta G(A^\alpha)}{\delta \alpha}\right)$$

10.2.3 The Glashow-Weinberg-Salam Theory

<u>Goal</u>: Generalize the Higgs mechanism to the Standard model

1) $\mathcal{L}_{EWS} = \mathcal{L}_{Fermion} + \mathcal{L}_{Yang-Mills} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$

2) Gauge symmetry:

$$\underbrace{SU(2)_L}_{\text{Weak isospin}} \times \underbrace{U(1)_Y}_{\text{Weak hypercharge}}$$

- $SU(2)_L \to$ 3 generators T^i, $i = 1,2,3$

$$[T^i, T^j] = i\,\varepsilon^{ijk}\,T^k$$

$\to$ Irreducible representations:

- 1D: Trivial rep $\hat{T}^i = 0$ (singlet rep)
- 2D: Pauli matrices $\hat{T}^i = \frac{\sigma^i}{2}$ (doublet rep) — in the following

$\to$ Eigenvalues of $\hat{T}^3$ = The weak isospin T^3

$$\begin{cases} T^3 = \pm\frac{1}{2} & \text{doublet} \\ T^3 = 0 & \text{singlet} \end{cases}$$

- $U(1)_Y \to$ 1 generator Y

$[Y, T^i] = 0$

Schur's lemma $\to$ $\hat{Y} = \text{Hypercharge } Y \times \mathbb{1}$