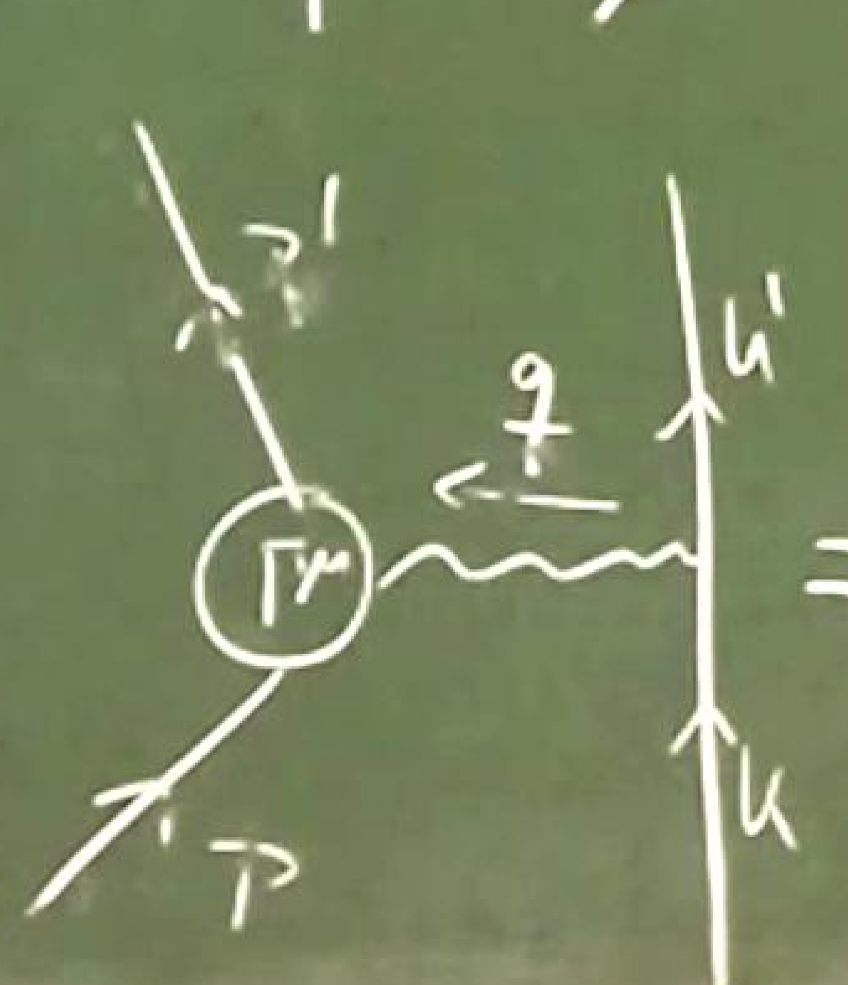
Recap:

6.3 Electron vertex function

6.3.1 Formal structure

1] $i\mathcal{M} =$  $+ \dots$

$=$  $= ie^2 \underbrace{\bar{u}(p')\Gamma^\mu u(p)}_{e} \frac{1}{q^2} \underbrace{\bar{u}(k)\gamma_\mu u(k)}_{\text{muon (heavy)}}$

$m \to \infty$

2] General form of $\Gamma^\mu(p,p')$

$$\Gamma^\mu(p,p') = f(p^\mu, p'^\mu, \gamma^\mu, m, e, c)$$

3] Restrictions

{ All equations below are valid if evaluated as $\bar{u}\,\Gamma^\mu\, u$!

i] Lorentz covariance

$$\Gamma^\mu = A\gamma^\mu + B(p'^\mu + p^\mu) + C(p'^\mu - p^\mu)$$

$\sum_s v_s \bar{v}_s$

ii] $(p^2, pp', p'^2, p_\mu\gamma^\mu = \not{p},)$

$(\not{p} - m)u = 0 \Rightarrow \boxed{\begin{matrix}\not{p}u(p) = m\,u(p) \\ \bar{u}\,\not{p}' = \bar{u}(p')\,m\end{matrix}}$

$$\Rightarrow X = X(p^\mu, p'^\mu, m, e, c)\cdot \mathbb{1}$$

$\in A, B, C$

III) $q^2 = (p' - p)^2 = 2(m^2 - pp')$

$$X = X(q^2, m, e, C)$$

IV) Ward identity: U(1) gauge-symmetry of QED

$$q_\mu \Gamma^\mu \overset{*}{=} 0$$

Noether's theorem for QFT

$$\rightarrow 0 = q_\mu \Gamma^\mu = A \underbrace{\overset{\cancel{q}}{q_\mu \gamma^\mu}}_{=0} + B \underbrace{q_\mu (p'^\mu + p^\mu)}_{=0} + C q^2 \Rightarrow C = 0$$

$\bar{u}(\cancel{p}' - \cancel{p}) u = (m - m)\bar{u} u = 0$ $p'^2 = m^2 = p^2$

4) Gordon identity:

$$\bar{u}(p') \frac{p'^\mu + p^\mu}{2m} u(p) \overset{\circ}{=} \bar{u}(p') \gamma^\mu u(p) - \bar{u}(p') \frac{i\sigma^{\mu\nu} q_\nu}{2m} u(p)$$

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

5) $$\boxed{\Gamma^\mu(p, p') = \underline{\gamma^\mu F_1(q^2)} + \frac{i\sigma^{\mu\nu} q_\nu}{2m} F_2(q^2)} = \underline{\gamma^\mu} + O(\alpha)$$

$F_i(q^2)$: Form factors

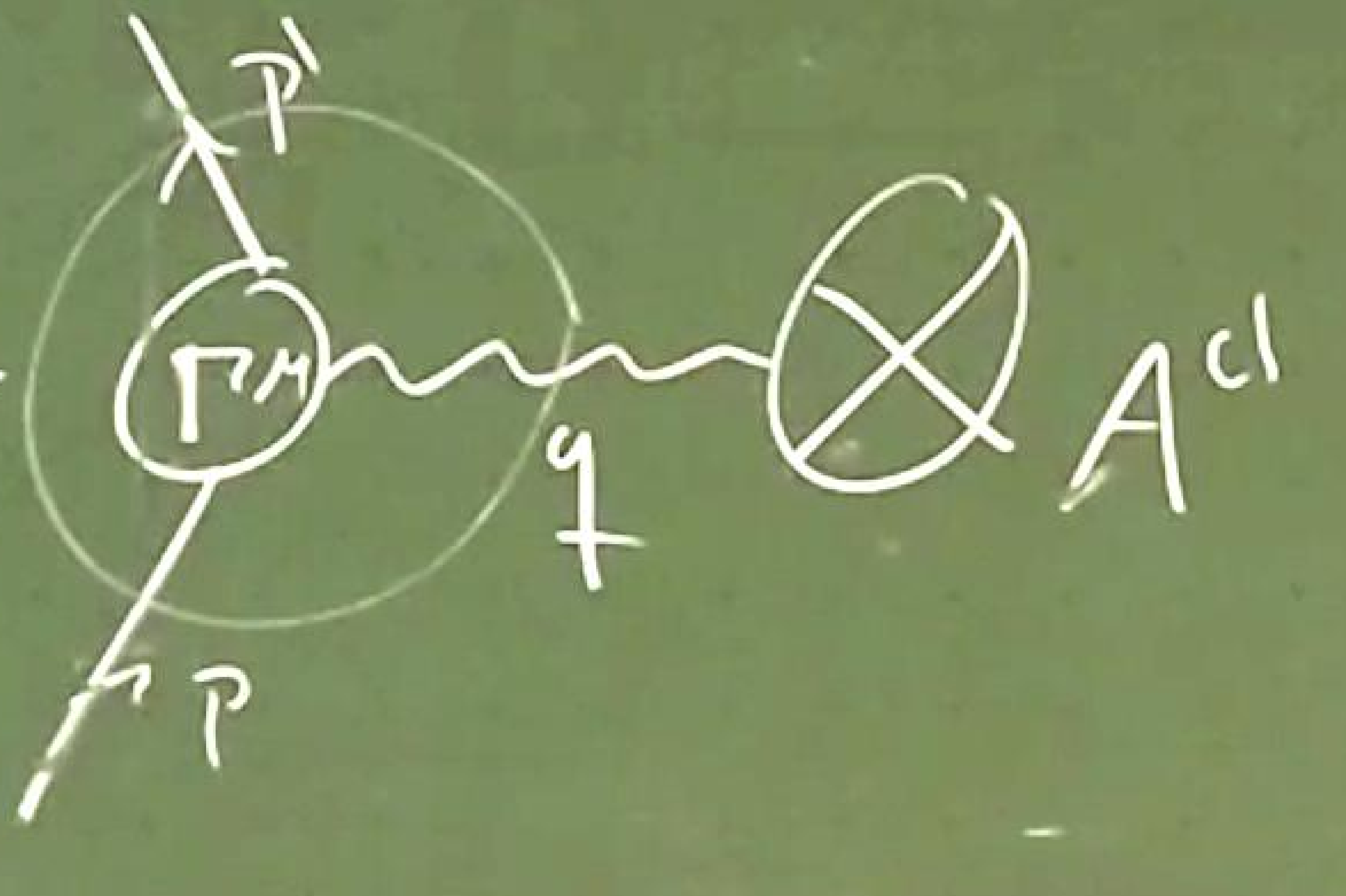
$$\underline{F_1(q^2) = 1 + O(\alpha)} \qquad \underline{F_2(q^2) = 0 + O(\alpha)}$$

6.3.2 Landé g-factor

1. Setting: $\exists$ classical external field $A_\mu^{cl}(x)$

$$H_{int} = e\int d^3x\, \bar{\Psi}(x)\gamma^\mu\Psi(x)A_\mu^{cl}(x)$$

$iM(2\pi)\delta(p'^0-p^0) =$



$q = p'-p$

2) Electric charge:

i) $A_\mu^{cl}(x) = (\phi(\vec{x}), 0,0,0) \Rightarrow A_\mu(q) = (2\pi\delta(q^0)\phi(\vec{q}), 0,0,0)$

ii) $iM = -ie\,\bar{u}(p')\Gamma^0(p',p)u(p)\cdot\phi(\vec{q})$

iii) $\phi(\vec{x})$ slowly varying $\rightarrow$ $\phi(\vec{q})$ concentrated $\vec{q}=0$

$\rightarrow$ take limit $\vec{q}\rightarrow 0$

$$iM \approx -ie\,\bar{u}\,F_1(0)\gamma^0 u(p)\cdot\phi(\vec{q})$$

$|\vec{p}| \ll m^2$

$$\approx \underbrace{-ie\,F_1(0)\phi(\vec{q})}_{\hat{V}(\vec{q})}\; 2m\,\xi'^\dagger\xi$$

iv) Born approximation with potential

$iM = -i\hat{V}(\vec{q})$

$V(\vec{x}) = eF_1(0)\phi(\vec{x})$

v) $\Rightarrow F_1(0) = 1$

$$F_1(0) = \underbrace{F_1^{(0)}(0)}_{} + \alpha F_1^{(1)}(0) + \alpha^2\ldots = 1 + O(\alpha)$$

$\rightarrow$ $\boxed{F_1^{(n)}(0) = 0 \quad \text{for } n\geq 1}$

3] Magnetic moment

i] $A^{cl}_\mu(x) = (0, \vec{A}(x))$

$\to A^{cl}_\mu(q) = (0, 2\pi\delta(q^0)\vec{A}(\vec{q}))$

ii] $iM = -ie\bar{u}\,\Gamma^\mu u\, A^{cl}_\mu(\vec{q})$

$= ie\bar{u}(p')\left[\underbrace{\gamma^i F_1(q^2) + \frac{i\sigma^{i\nu}}{2m} q_\nu F_2(q^2)}_{q=0,\ |\vec{p}|^2 \ll 1}\right] u(p)\, A^i_{cl}(\vec{q})$

iii] $\bar{u}(p')\gamma^i u(p) \overset{\circ}{=} \boxed{\frac{p^i + p'^i}{2m}}\, 2m\,\xi'^\dagger\xi + 2m\,\xi'^\dagger\left(\frac{-i}{2m}\epsilon^{ijk} q^j \sigma^k\right)\xi$

$(p - eA)^2 = pA + Ap$

iv] $\frac{iq_\nu}{2m}\bar{u}(p')\sigma^{i\nu}u(p) \overset{\circ}{\approx} 2m\,\xi'^\dagger\left(\frac{-i}{2m}\epsilon^{ijk} q^j \sigma^k\right)\xi$

v] Summary

$iM \overset{q\to 0}{\approx} -ie\,\xi'^\dagger\left\{\frac{-1}{2m}\sigma^k\left[F_1(0) + F_2(0)\right]\right\}\xi \cdot \underbrace{\left[-i\epsilon^{ijk} q^j A^i_{cl}(\vec{q})\right]}_{\tilde{B}^k_{cl}(\vec{q})}(2m)$

vi] Born approximation. $\vec{B}_{cl} = \nabla \times \vec{A}_{cl}$

$V(\vec{x}) = -\langle\vec{\mu}\rangle \cdot \vec{B}$

$\langle\vec{\mu}\rangle = \frac{e}{2m}\left[F_1(0) + F_2(0)\right]\xi'^\dagger\frac{\sigma^k}{2}\xi$

$= g \cdot \boxed{\mu_B}\,\langle\vec{S}\rangle$, $\quad \mu_B = \frac{e}{2m}$

$\Rightarrow$ Landé factor:

$g = 2[\overset{1}{F_1(0)} + F_2(0)]$

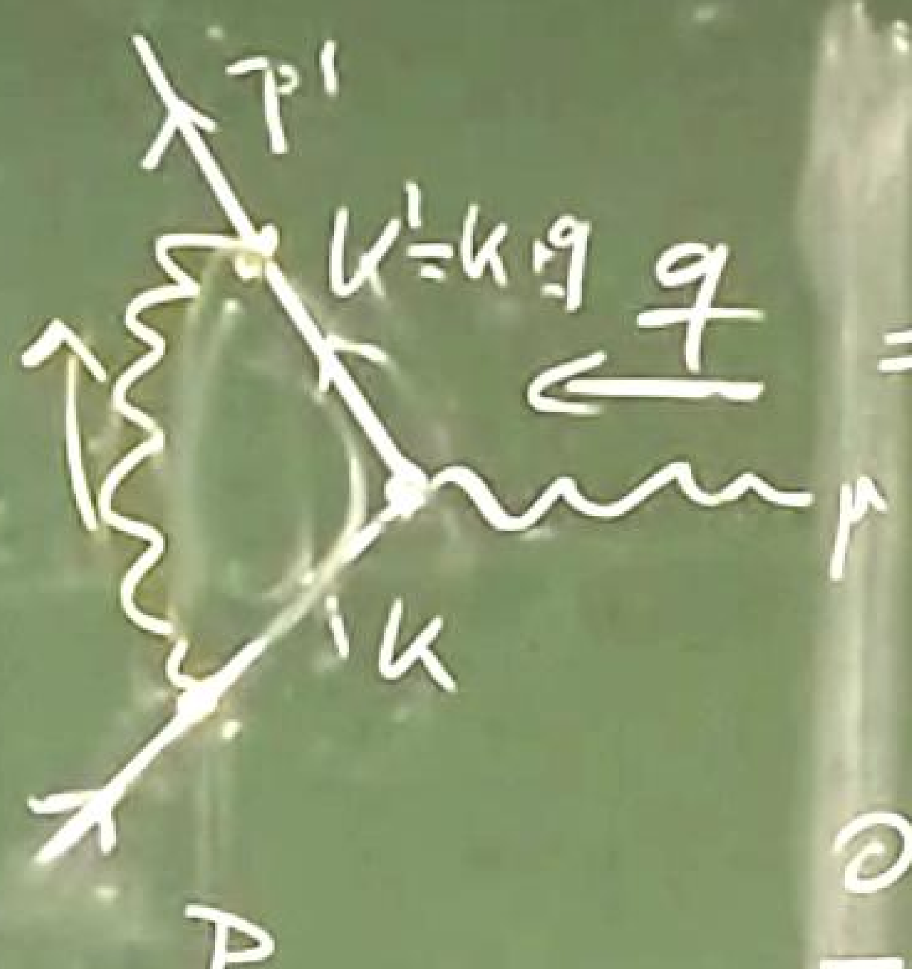
$= 2 + 2F_2(0)$

$= \underbrace{2}_{\text{Dirac equation}} + \underbrace{2\alpha F_2^{(1)}(0)}_{\text{Anomalous magnetic moment}} + O(\alpha^2)$

6.3.3 Evaluation

1. Scattering amplitude:

$$\bar u(p')\,[\Gamma^{(1)}(p',p)]^\mu\, u(p)$$



$$= \int \frac{d^4k}{(2\pi)^4}\,\frac{-ig_{\nu\rho}}{\tilde q^2+i\epsilon}\,\bar u(p')(-ie\gamma^\nu)\,\frac{i(\not k'+m)}{k'^2-m^2+i\epsilon}\,\gamma^\mu\,\frac{i(\not k+m)}{k^2-m^2+i\epsilon}\,(-ie\gamma^\rho)\,u(p)$$

$\tilde q = p-k$

$$\overset{\circ}{=} 2ie^2\int\frac{d^4k}{(2\pi)^4}\,\frac{\bar u(p')\left[\not k\,\gamma^\mu \not k' + m^2\gamma^\mu - 2m(k+k')^\mu\right]u(p)}{(\underbrace{\tilde q^2+i\epsilon}_{A_1})(\underbrace{k'^2-m^2+i\epsilon}_{A_2})(\underbrace{k^2-m^2+i\epsilon}_{A_3})}$$

2) Feynman Parameter:

$$\boxed{\frac{1}{A_1\cdots A_n} = \left(\prod_{i=1}^{n}\int_0^1 dx_i\right)\delta\left(\sum_{i=1}^{n}x_i-1\right)\frac{(n-1)!}{[\underbrace{x_1A_1+\dots+x_nA_n}_{D}]^n}}$$

x_i: Feynman parameters

3) $$\frac{1}{A_1A_2A_3} = \int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)\,\frac{2}{D^3}$$

$$D \overset{\circ}{=} k^2+2k(yq-zp)+yq^2+zp^2-(x+y)m^2+i\epsilon$$

$$= \underbrace{l^2}_{k+yq-zp} - \underbrace{\Delta}_{-xyq^2+(1-z)^2m^2>0} + i\epsilon$$

4) $\bar{u}(p')\,[\;\text{—"—}\;]\,u(p)$

$$\approx u(p')\left\{-\frac{1}{2}\gamma^\mu \ell^2 + [-y\not{q} + z\not{p}]\,\gamma^\mu\,[(1-z)\not{q} + z\not{p}] + m^2\gamma^\mu - 2m\,[(1-2y)\,q^\mu + 2z\gamma^\mu]\right\}u(p)$$