Recap

H_{int} in interaction picture

$$\langle\Omega|T\phi(x)\phi(y)|\Omega\rangle = \lim_{T\to\infty(1-i\epsilon)} \frac{\langle 0|T\{\phi_I(x)\phi_I(y)\exp[-i\int_{-T}^{T}dt\,H_I(t)]\}|0\rangle}{\langle 0|T\{\exp[-i\int_{-T}^{T}dt\,H_I(t)]\}|0\rangle}$$

Wick's Theorem

All full contractions

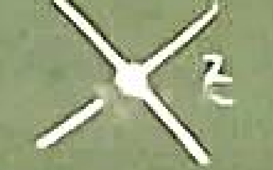
Feynman propagators

Many equivalent terms

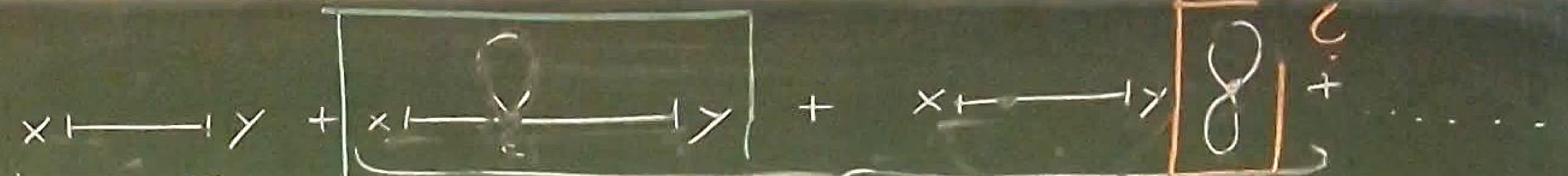
Feynman Diagrams

$$= \sum{}' \left\{ \text{Feynman diagrams with two external points } x \text{ and } y \right\}$$

Feynman diagram $\xrightarrow{\text{Feynman rules}}$ Analytic expression

1. x ——— y $= D_F(x-y)$
2.  $= (-i\lambda)\int d^4z$
3. x |—— $= 1$
4. Divide by sym. factor $\frac{1}{S}\times$

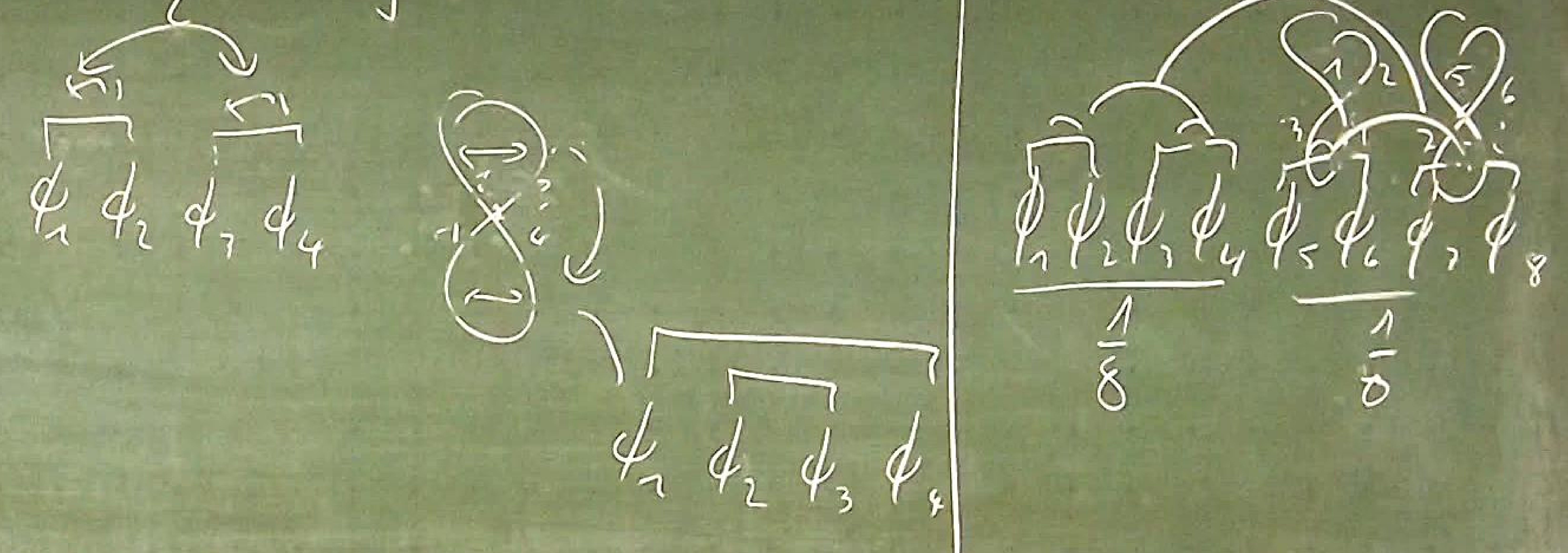
Example:

$\langle\Omega|T\phi(x)\phi(y)|\Omega\rangle =$ 

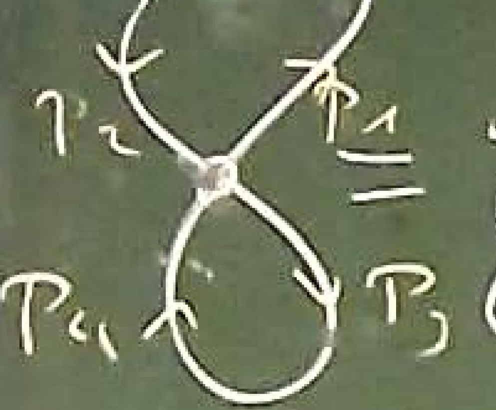
0th order — 1st order

$\boxed{\text{— " —}}$?

$$= \frac{1}{2}(-i\lambda)\int d^4z\, D_F(x-z)\, D_F(z-z)\, D_F(z-y)$$



7| Problem: Disconnected pieces of diagrams diverge!



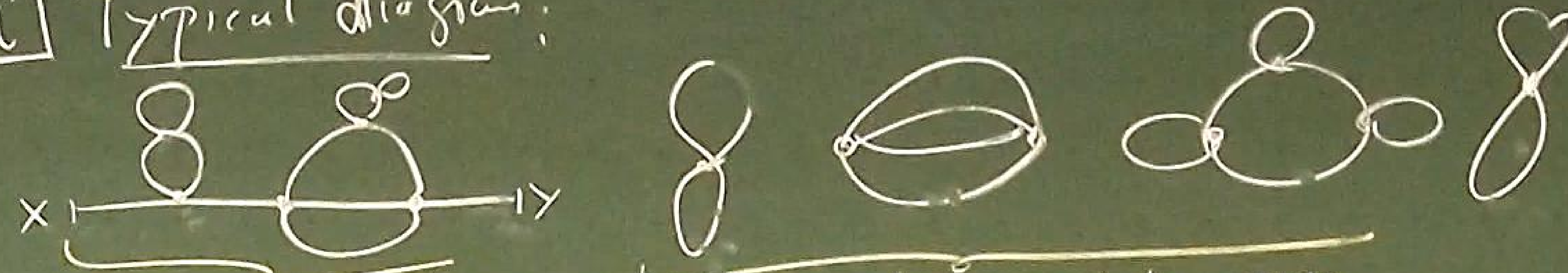
$$= \frac{1}{8} (-i\lambda) \int d^4z \, \underbrace{D_F(z-z)\, D_F(z-z)}_{\text{const.}}$$

$$\int d^4z \to \int_{-T}^{T} dt \int d^3x \to 2T \cdot (\text{volume of space})$$

$$\to \delta(\cancel{p_2} + \cancel{p_4} - \cancel{p_1} - \cancel{p_3}) = \delta(0)$$

8| Exponentiation of disconnected diagrams

i) Typical diagram:



ii) $V = \{V_1, V_2, \ldots\} = \{$ Disconnected Feynman diagrams without external points $\}$

$F_{xy} = \{$ Connected Feynman diagrams with external points x, y $\}$

$\to$ Feynman diagram $F = \{ F^{xy}, \underbrace{V_1, V_1, \ldots V_1}_{n_1}, \underbrace{V_2, \ldots V_2}_{n_2}, V_3, \ldots \}$

$$F = F^{xy} \cdot \prod_i \frac{1}{n_i!} (V_i)^{n_i}$$

iv) $\langle 0 | T\{\phi(x)\phi(y)\, e^{-i\int dt\, H_I(t)}\} | 0 \rangle$

$$= \sum_{F \in F^{xy}} \sum_{n_1, \ldots, n} \left[F \cdot \prod_i \frac{1}{n_i!} (V_i)^{n_i} \right]$$

$$= \left[\sum_{F \in F^{xy}} F \right] \times \left[\sum_{n_1 \ldots} \prod_i \frac{1}{n_i!} (V_i)^{n_i} \right]$$

$$= \text{—〃—} \left[\prod_i \sum_{n_i} \frac{1}{n_i!} (V_i)^{n_i} \right]$$

$$= \text{—〃—} \exp\left[\sum_i V_i \right]$$

$$\langle 0 | T\{\phi(x)\phi(y)\, e^{-i\int dt\, H_I(t)}\} | 0 \rangle$$

$$= \sum \left(\bar{F}^{xy} \right) \cdot e^{\sum (V)}$$

9) Denominator:

$$\langle 0 | T\{ e^{-i\int dt\, H_I(t)} \} | 0 \rangle = e^{\sum (V)}$$

10) Two-point correlator:

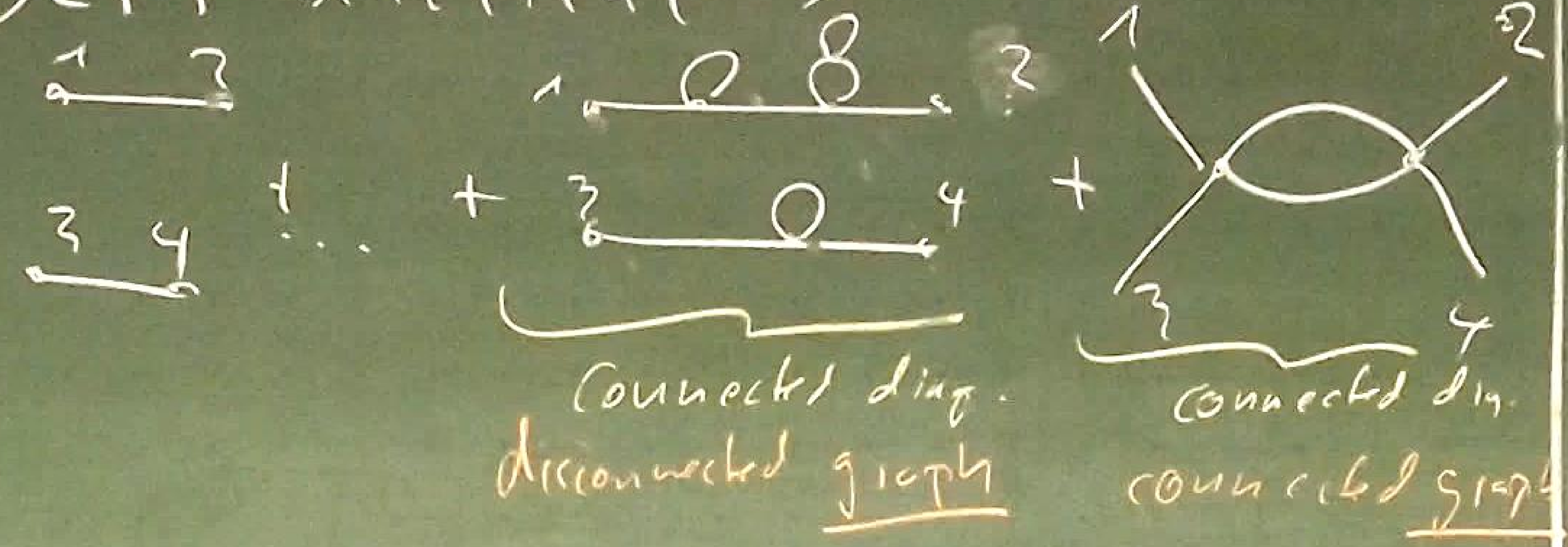
$$\boxed{\langle \Omega | T\{\phi(x)\phi(y)\} | \Omega \rangle = \sum \left(\bar{F}^{xy} \right) = \left\{ \begin{array}{l} \text{Sum of all } \underline{\text{connected}} \\ \text{diagrams with } \underline{\text{two}}^{\,n} \text{ external} \\ \text{points} \end{array} \right\}}$$

$\bar{F}^{x_1 \dots x_4}$

$\phi(x_1)\phi(x_2)\dots\phi(x_4)$

Note 4.1

- $\langle \Omega | T\, \phi_1 \phi_2 \phi_3 \phi_4 | \Omega \rangle$



- Disconnected diagrams = "Vacuum bubbles"

- Interpretation:

$$\lim_{T \to \infty(1-i\epsilon)} \langle 0 | T\, \phi(x)\phi(y) \exp\left[-i \int_{-T}^{T} dt\, H_I(t) \right] | 0 \rangle$$

$$= \underbrace{\langle \Omega | T\, \phi(x)\phi(y) | \Omega \rangle}_{\|} \times \lim_{T \to \infty(1-i\epsilon)} |\langle 0 | \Omega \rangle|^2 e^{-iE_0 2T}$$

$$= \sum \left(\bar{F}^{xy} \right) \cdot e^{\sum (V)}$$

Vacuum energy density

$$\sum V_i = \sum (V) = -i E_0\, 2T$$

$$V_i = \tilde{V}_i \cdot (2T \cdot V)$$

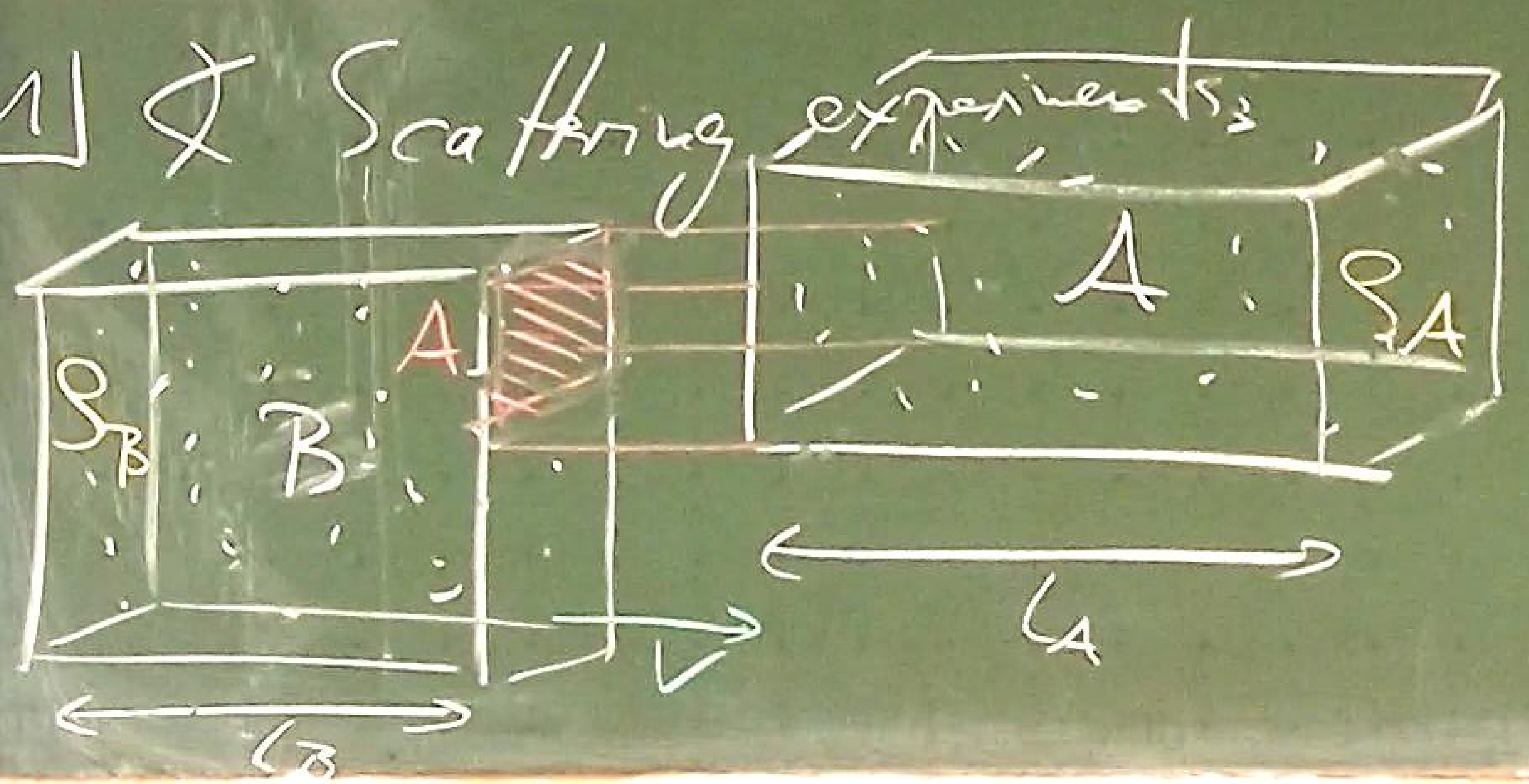
$$\Rightarrow \boxed{\frac{E_0}{V} = i \sum \tilde{V}_i}$$

→ Vacuum bubbles determine the vacuum energy density

4.5 Cross Sections and the S-Matrix

The Cross section

1] ☆ Scattering experiments:

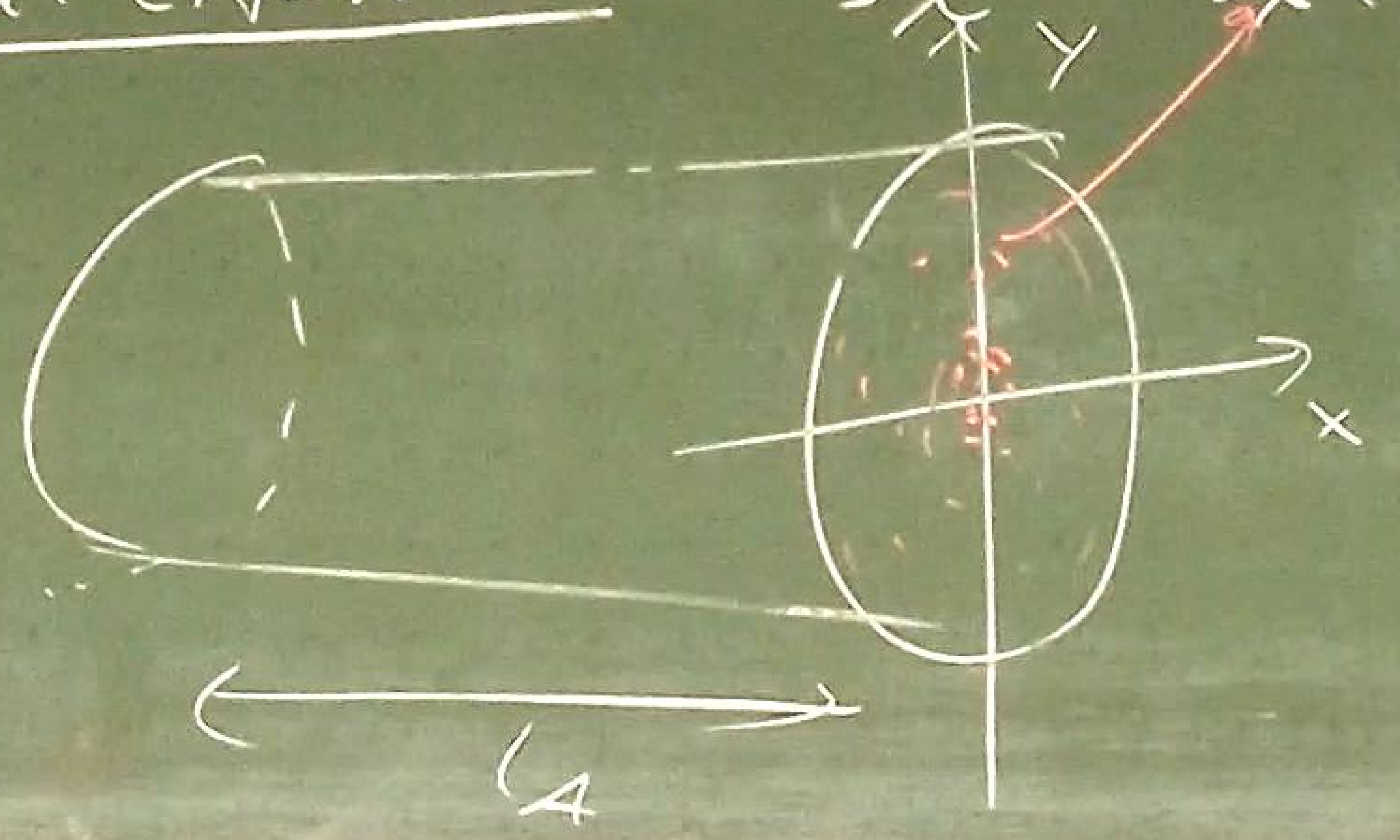


2] Cross section:

$$\sigma_X \equiv \frac{\#\text{scattering events (with outcome } X)}{\rho_A l_A \rho_B l_B A}$$

$$[\sigma_X] = L^2 = \text{Area}$$

3] Real experiments: $\rho_X \to \rho_X(x,y)$



$$\#\text{scattering events }(X) = \sigma_X \cdot l_A l_B \int dx\,dy\, \rho_A(x,y)\,\rho_B(x,y)$$

4] Many outcomes possible.

$$e^+e^- \to \left\{ \begin{array}{l} e^+e^- \\ \mu^+\mu^- \\ \mu^+\mu^-\gamma \\ \vdots \end{array} \right\} X$$

5] Differential cross section

* Scattering outcome X of n final particles with moment $(\vec{p}_1, \dots, \vec{p}_n) \in V_p \subset \mathbb{R}^{3n}$

$$\sigma_{X|V_p} = \int_{V_p} d^3p_1 \dots d^3p_n \underbrace{\frac{d\sigma}{d^3p_1 \dots d^3p_n}}_{\text{Differential cross section}}$$

→ Constrained by 4-momentum conservation. $\sum_i p_i = \text{const.}$

Special case: $n=2$

→ 6 dof $(\vec{p}_1, \vec{p}_2)$ + 4 constraints

→ 2 dof

→ Scattering directions (φ, ϑ) in the centre-of-mass frame

$$\frac{d\sigma}{d^3p_1 d^3p_2} \longrightarrow \boxed{\frac{d\sigma}{d\Omega}}$$

$d\Omega = \sin\vartheta \, d\vartheta \, d\varphi$