Organization

* www.itp3.uni-stuttgart.de/teaching/qft22
* 1 Problemset per week, written + oral exercises

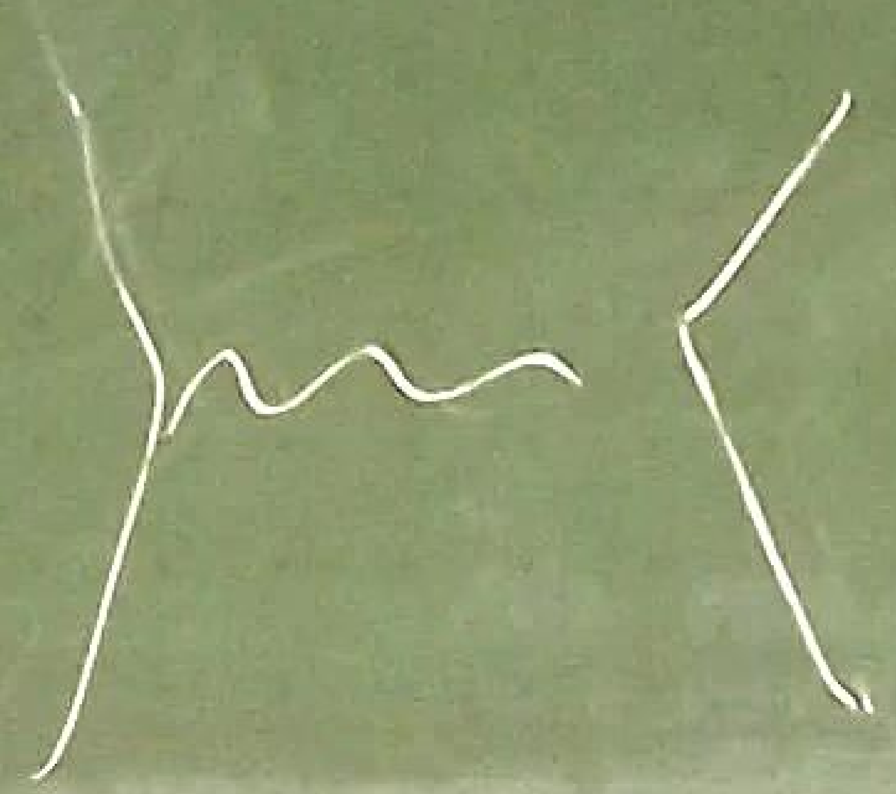
80% 66%

2 x blackboard presentations

Sign up for exercises:

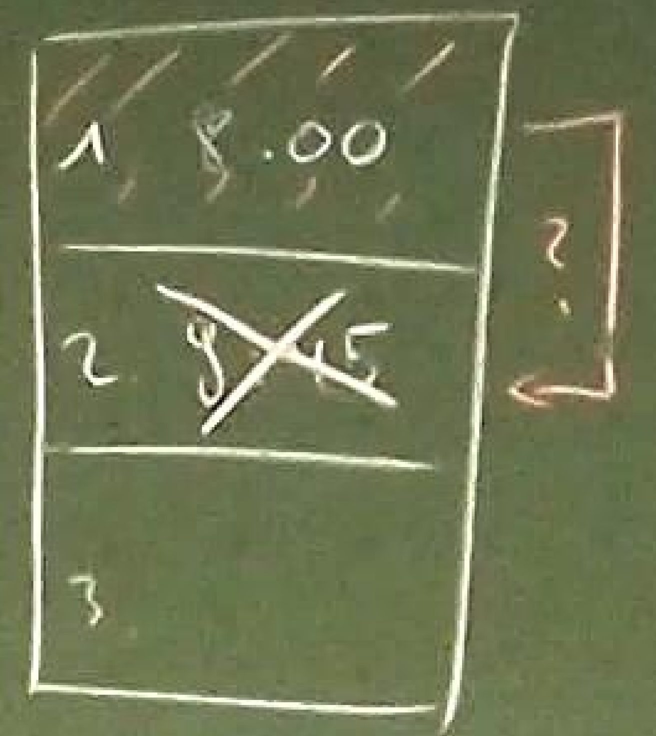
=> lms.itp3.uni-stuttgart.de

Lecture Key: $\boxed{\text{qftSS22!}}$

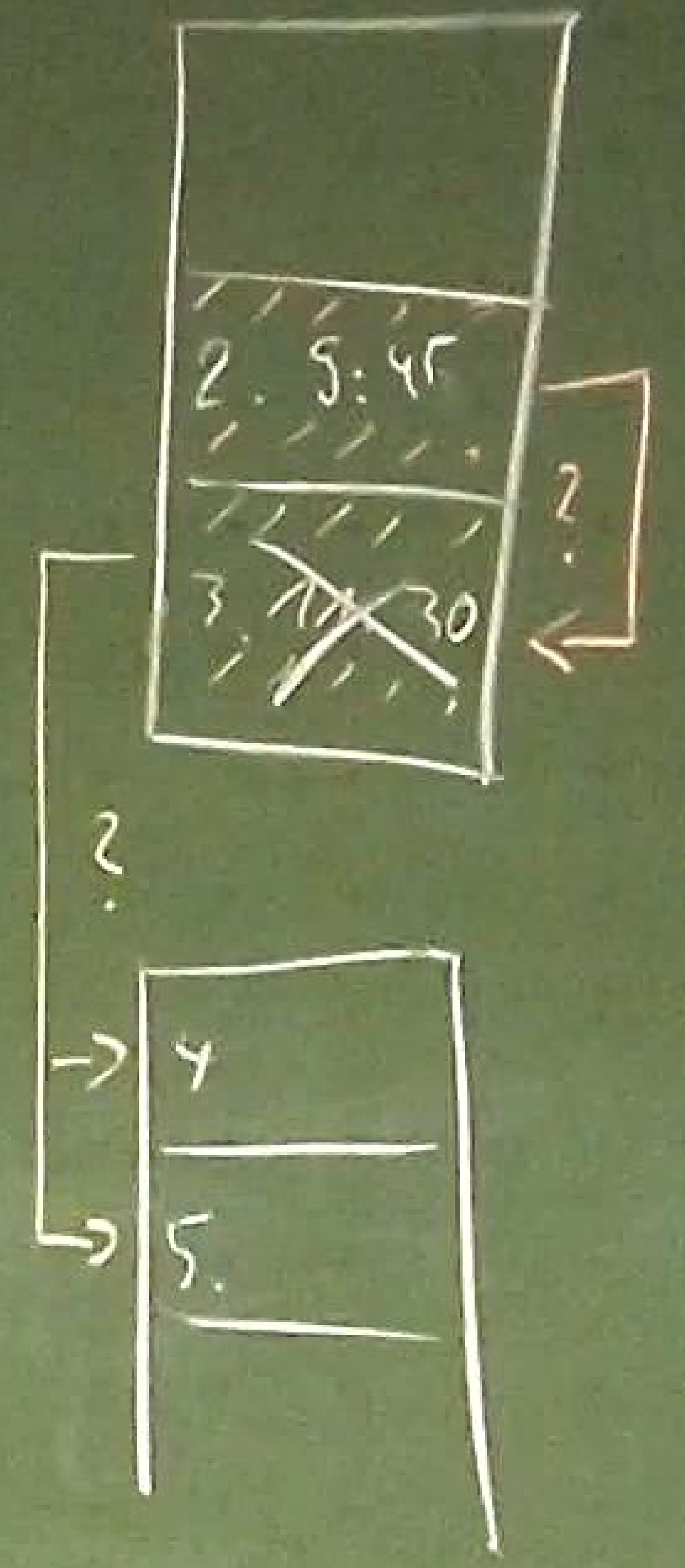


* 2 lectures per week:

Wed



Fri



* Based on Peskin & Schroeder

* Requirements:

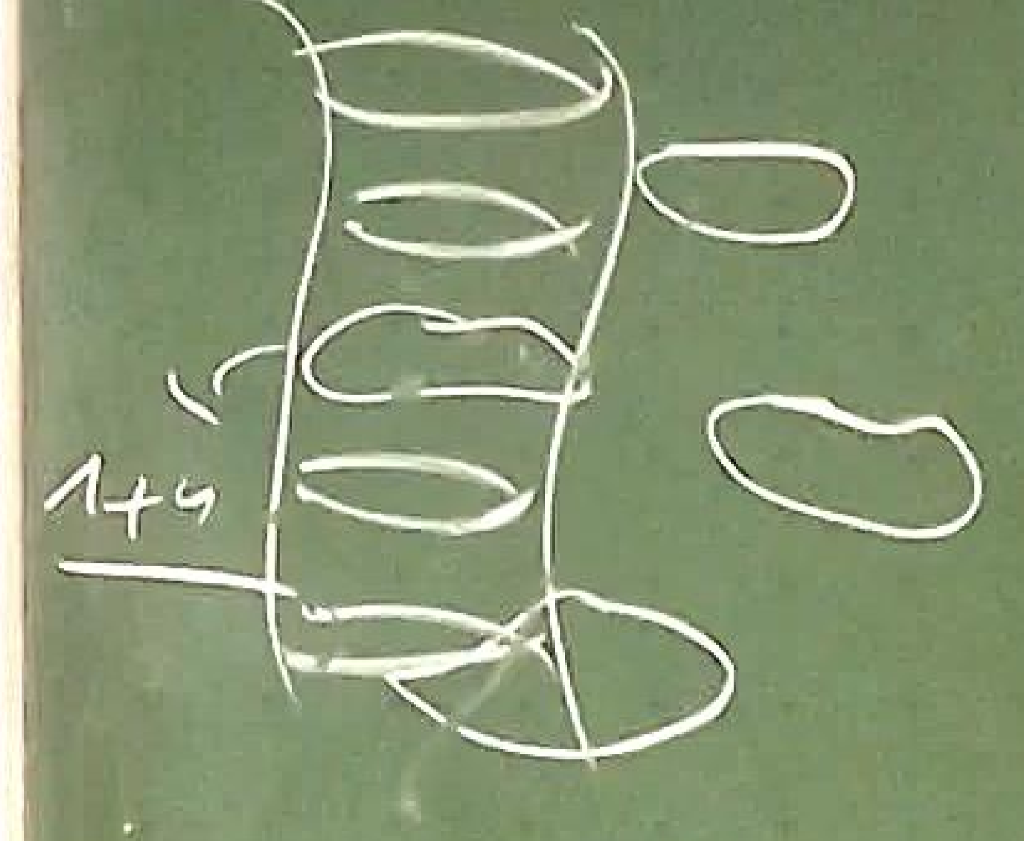
- QM (second quantization)
- SRT (tensor calculus)
- Complex analysis (residue theorem)

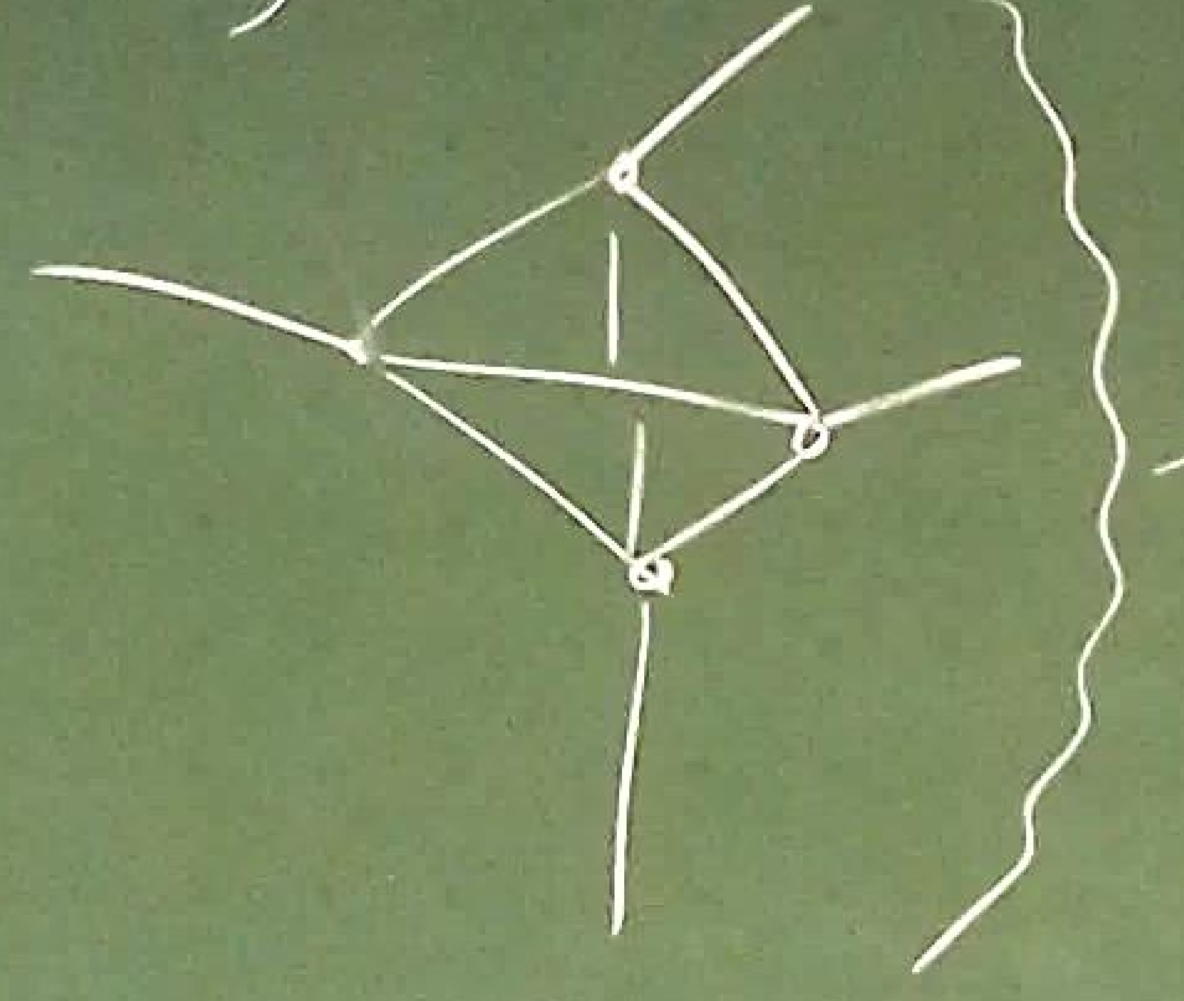
? ? ?

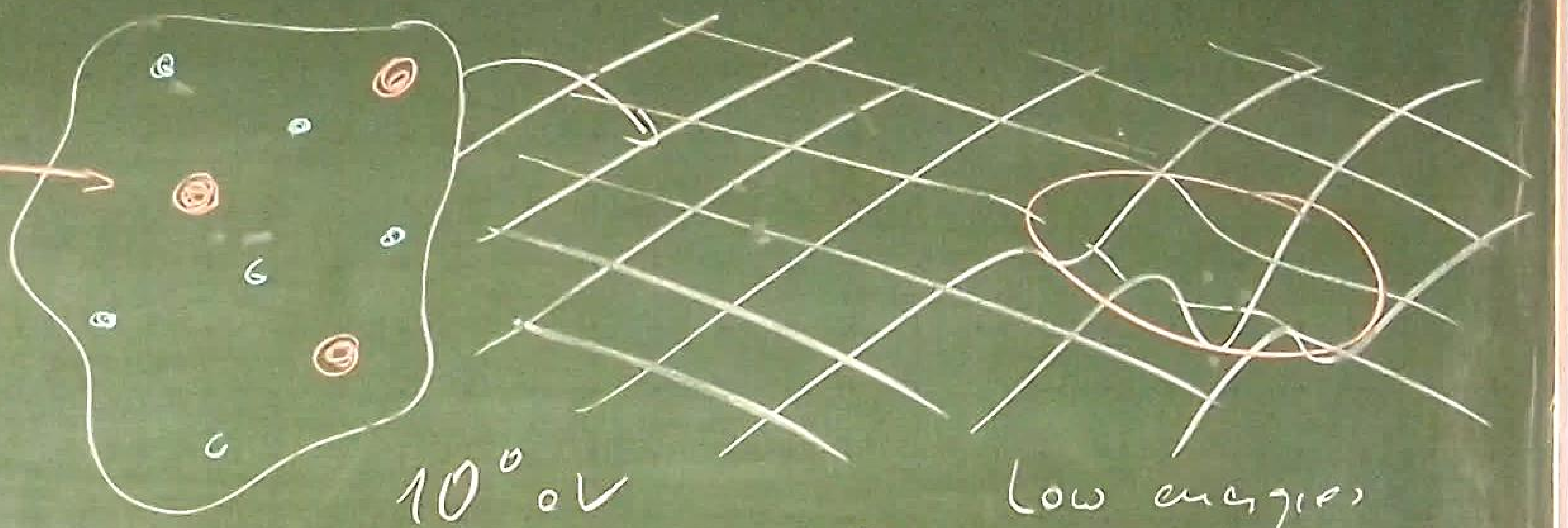
High energy physics

Condensed matter physics

Elementary what ?? (Strings, Spin-foam... ?) → Emergent Quantum fields = Elementary Quantum fields (Dirac fields, Gauge fields...) → Emergent particles = Elementary particles (Electrons, Phonons...) → Emergent Quantum fields (Magnetization...) → Emergent Quasiparticles (Magnons...)







High energies
Small lengths
Short times

10^{28} eV (Planck)

Standard Model

10^{9} eV

10^{0} eV

Low energies
Large lengths
Long times

This course

1. Elements of classical field theory

1.1. Lagrangian and Hamiltonian formalism

Recap: Classical mechanics of "point"

1. DOF q_i $i = 1 .. N$
2. Lagrangian $L(\{q_i\}, \{\dot{q}_i\}, t)$ $\dot{q}$
   $= T - V$
3. Action $S[q] = \int dt\, L(q(t), \dot{q}(t), t)$
   $q(t)$
4. Hamilton's principle of least action
   $$\frac{\delta S[q]}{\delta q} \stackrel{!}{=} 0 \Leftrightarrow \delta S = \int dt\, \delta L \stackrel{!}{=} 0$$
5. Euler-Lagrange equations $(i = 1 .. N)$
   $$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = 0$$

Analogous: Lagrangian Field Theory

1. $\cancel{x}$ $\phi(x)$ on spacetime $x \in \mathbb{R}^{1,3}$ $\in \mathbb{R}^4$
   with $\partial_\mu \phi(x)$
   $\partial_0 = \partial_t$
   ∂_i
   $x\, y\, t$
2. Lagrangian density
   $\mathcal{L}(\phi(x), \partial\phi, x)$

$\rightarrow$ Lagrangian:

$$L = \int d^3x\, \mathcal{L}(\phi, \partial\phi)$$

3. Action

$$S[\phi] = \int dt\, L = \int d^4x\, \mathcal{L}(\phi(x), \partial\phi(x))$$

4. Action principle:

$$0 \overset{!}{=} \delta S = \int d^4x\, \delta\mathcal{L}$$

$$= \int d^4x \left\{ \frac{\partial\mathcal{L}}{\partial\phi}\delta\phi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta(\partial_\mu\phi) \right\}$$

$$= \int d^4x \left\{ \quad - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right)\delta\phi + \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta\phi\right) \right\}$$

$$= \int d^4x \underbrace{\left\{ \frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right) \right\}}\delta\phi$$

$$+ \int_\partial dA \left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} \underset{0}{\underset{\shortparallel}{\delta\phi}} \right) \overset{!}{=} 0$$

5. Euler Lagrange equations.

$$\boxed{\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right) = 0}$$

Recap Hamiltonian Mechanics

Lag.

$$L(q,\dot q,t) \xrightarrow{\text{Legendre transformation}}$$

$$p \equiv \frac{\partial L}{\partial \dot q} \Leftrightarrow \dot q = \dot q(p)$$

$$H(q,p,t) = p\dot q - L(q,\dot q,t)$$

Analogous: Hamiltonian Field Theory

1. $\dot{x}$ $\quad x = x_i \hat{=} i$

$$\frac{\partial L}{\partial \dot{q}_i} = p_i \hat{=} p(x) = \frac{\partial L}{\partial(\dot{\phi}(x))}$$

$$= \frac{\partial}{\partial \dot{\phi}(x)} \int d^3y\, \mathcal{L}(\phi(y), \dot{\phi}(y))$$

$$= \sum_y d^3y \frac{\partial}{\partial \dot{\phi}(x)} \mathcal{L}$$

$$\delta_{x,y} \frac{\partial \mathcal{L}}{\partial \dot{\phi}}\Big|_{x=y} = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}\, d^3x$$

$$= \frac{\partial \mathcal{L}}{\partial \dot{\phi}(x)},$$

$\equiv \pi(x)$ Momentum density conjugate of ϕ

② Hamiltonian:

$$H = \sum_x \underbrace{p(x)}_{\pi(x)\cdot d^3x} \dot{\phi}(x) - \sum_x \mathcal{L}(\phi(x), \dot{\phi}(x))\, d^3x$$

$d^3x \to 0$

$$= \int d^3x \underbrace{\{\pi(x)\dot{\phi}(x) - \mathcal{L}\, \phi(x), \dot{\phi}(x)\}}_{\text{Hamiltonian density } \mathcal{H}(\phi, \pi)}$$

Example 1.1. Free scalar field

1. $\phi : \mathbb{R}^3 \times \mathbb{R} \longrightarrow \mathbb{R}$

$\phi(x) = \phi(\vec{x}, t)$

2. $\mathcal{L}$-density:

$$\mathcal{L} = \frac{1}{2}(\partial_t \phi)^2 - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}m^2\phi^2$$

$$= \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2$$

$(\partial_\mu\phi)(\partial^\mu\phi)$

3. Interpretation:

4. EOM

$$-m^2\phi - \partial_\mu(\partial^\mu\phi) = 0$$

$$\Leftrightarrow \boxed{(\partial_\mu\partial^\mu + m^2)\,\phi = 0}$$

(classical) Klein-Gordon equation

5. $$\pi = \frac{\partial\mathcal{L}}{\partial\dot\phi} = \dot\phi$$

6. $$\mathcal{H} = \pi\dot\phi - \frac{1}{2}\dot\phi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi$$

$$= \frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2$$

1.2 Symmetries and Conservation Laws

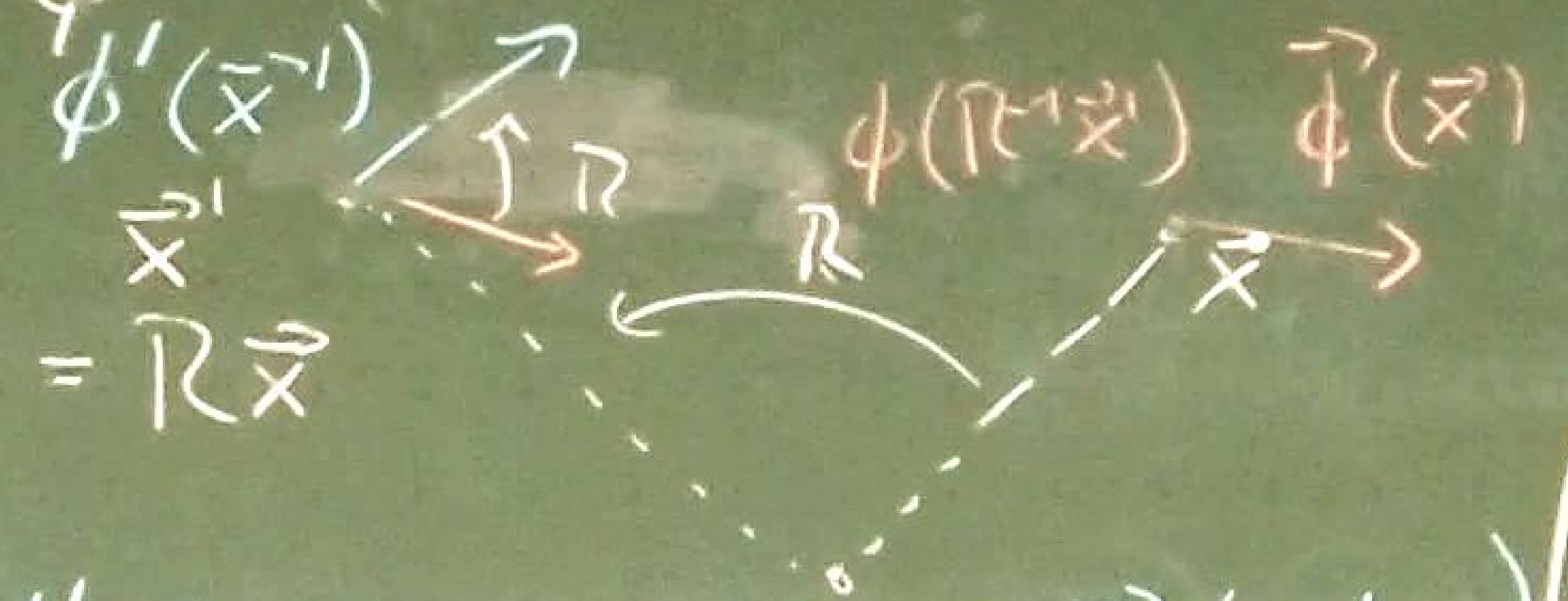
1.$\cancel{2}$ Transformations of coordinates and fields.

$x \mapsto x' = x'(x)$ and

$\phi(x) \mapsto \phi'(x') = \mathcal{F}(\phi(x))$

Example 1.2 Rotation of a vector field $\vec\phi$

i) $\vec\phi = (\phi_1, \phi_2, \phi_3)$, $R \in SO(3)$



$$\vec\phi'(\vec x') = R\vec\phi(\vec x) = R\vec\phi(R^{-1}\vec x') \begin{pmatrix} S(x) \\ T(x) \end{pmatrix}$$

2. $$S' \equiv S[\phi'] = \int d^dx'\, \mathcal{L}(\phi'(x'), \partial'_\mu\phi'(x'))$$

$$= \int d^dx'\, \mathcal{L}(\mathcal{F}(\phi(x)), \partial'_\mu\mathcal{F}(\phi(x)))$$

$$= \int d^dx \left|\frac{\partial x'}{\partial x}\right| \mathcal{L}\left(\mathcal{F}(\phi(x)), \frac{\partial x^\nu}{\partial x'^\mu}\partial_\nu\mathcal{F}(\phi(x))\right)$$

Example 1.3 Translation:

1. $x' := x + a$

$\phi'(x') := \phi(x) = \phi(x'-a)$ scalar field

2. $\mathcal{F} = \mathbb{1}$ trivial

- $\phi'(x') = \mathcal{F}(\phi(x)) = \phi(x(x'))$
- $\frac{\partial x^\nu}{\partial x'^\mu} = \delta^\nu_\mu$

3. $S[\phi'] = \int d^dx\, \mathcal{L}(\phi'(x), \partial_\mu \phi'(x)) = \int d^dx\, \mathcal{L}(\phi(x), \partial_\mu \phi(x)) = S[\phi]$

Example 1.4 Scale transformations

scaling dimension

1. $x' = \lambda x$

$\phi' := \lambda^{-\Delta} \phi(x)$

2. $\mathcal{F}(\phi) = \lambda^{-\Delta}\phi$, $\frac{\partial x^\nu}{\partial x'^\mu} = \lambda^{-1}\delta^\nu_\mu$

$\hookrightarrow \left|\frac{\partial x'}{\partial x}\right| = \lambda^d$

3. Action:

$\boxed{S[\phi']} = \lambda^d \int d^dx\, \mathcal{L}(\lambda^{-\Delta}\phi(x), \lambda^{-1-\Delta}\partial_\mu\phi)$

$\overset{\mathcal{L}\ \mathrm{FSF}}{\underset{m=0}{=}} \lambda^{d-2-2\Delta} \int d^dx\, \mathcal{L}(\phi(x), \partial_\mu\phi(x)) = \boxed{\lambda^{d-2-2\Delta} S[\phi]}$

$\rightarrow S' = S$ iff $\Delta = \frac{d}{2} - 1$

Example 1.5. Phase rotation

1. $x' := x$

$\phi'(x') := e^{i\theta} \phi(x)$

2. $\mathcal{F}(\phi) = e^{i\theta} \phi$

$\frac{\partial x^\nu}{\partial x^\mu} = \delta^\nu_\mu, \quad \left|\frac{\partial x'}{\partial x}\right| = 1$

$\mathcal{L} = \phi \phi^* - (\nabla \phi)(\nabla \phi^*)$