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Problem 8.1: Coriolis force

[**Oral** | 2 pt(s)]

ID: ex_coriolis_force:km25

Learning objective

In this task, the influence of the Coriolis force on a falling stone is investigated.

Consider a stone falling from the Stuttgart television tower (height: 217 m). At the beginning, the stone is at rest. The influence of the centrifugal force is small and can be neglected. Set up the equations of motion. Write the solution as $\boldsymbol{r}(t) = \boldsymbol{r}_0(t) + \boldsymbol{r}_1(t) + \mathcal{O}(\Omega^2)$, where $\boldsymbol{r}_0(t)$ is the solution without Coriolis force and $\boldsymbol{r}_1(t)$ is the part of the solution that is linear in the angular velocity of the Earth Ω. Develop the equations of motion in the angular velocity. How far does the position of the stone deviate from free fall when it hits the ground? How does the result depend on the latitude?

Problem 8.2: Foucault pendulum

[**Written** | 4 pt(s)]

ID: ex_foucault_pendulum:km25

Learning objective

A Foucault pendulum can be used to demonstrate the rotation of the earth in a clear way and without astronomical observations. In this task, the trajectory of the pendulum is determined.

Consider an ideal pendulum of length l in an inertial system K that oscillates with a small deflection.

a) Show that the force acting on the pendulum mass can be written for small deflections as $\boldsymbol{F} = -m\omega_p^2(x\boldsymbol{e}_x + y\boldsymbol{e}_y)$. What is ω_p? **1**pt(s)

Now consider the rotation of the earth. We want to determine the movement of the pendulum in the coordinate system $K\prime$ of a person on earth. The coordinate system is chosen so that the z-axis is perpendicular to the earth's surface.

b) Set up the equation of motion in the rotating coordinate system. To do this, make sure that for realistic pendulum lengths l the angular velocity of the pendulum is much greater than the rotational velocity of the earth. How long would the pendulum have to be for the angular velocity of the pendulum and the rotational velocity of the earth to be the same? Use this to show that the equations of motion take the following form as a first approximation **1**pt(s)

$$\ddot{x} = \ 2\Omega_z \dot{y} - \omega_p^2 x, \tag{1a}$$
$$\ddot{y} = -2\Omega_z \dot{x} - \omega_p^2 y. \tag{1b}$$

What does Ω_z look like?

c) Use the approach $u_\pm = x \pm iy$ to decouple the equations of motion for x and y and find the general solutions $u_\pm(t)$ of the resulting differential equations. Show that for the initial conditions $x(0) = x_0$, $y(0) = y_0$ and $\dot{x}(0) = \dot{y}(0) = 0$ the following solution for x and y results **1pt(s)**

$$\begin{aligned} x(t) &= x_0 \cos(\Omega_z t) \cos(\omega_p t) + y_0 \sin(\Omega_z t) \cos(\omega_p t) \\ &\quad - y_0 \frac{\Omega_z}{\omega_p} \cos(\Omega_z t) \sin(\omega_p t) + x_0 \frac{\Omega_z}{\omega_p} \sin(\Omega_z t) \sin(\omega_p t), \end{aligned} \tag{2a}$$

$$\begin{aligned} y(t) &= y_0 \cos(\Omega_z t) \cos(\omega_p t) - x_0 \sin(\Omega_z t) \cos(\omega_p t) \\ &\quad + x_0 \frac{\Omega_z}{\omega_p} \cos(\Omega_z t) \sin(\omega_p t) + y_0 \frac{\Omega_z}{\omega_p} \sin(\Omega_z t) \sin(\omega_p t). \end{aligned} \tag{2b}$$

d) The coordinate system is chosen so that the pendulum is only deflected in the x-direction at the beginning. What are the values of $r(t) = \sqrt{(x(t))^2 + (y(t)^2)}$ and $\Theta = \arctan(y(t)/x(t))$ at the points in time $t_n = \pi/\omega_p n$ with $n \in \mathbb{N}$? **1pt(s)**

Now consider a pendulum at the north pole. Sketch the movement of the pendulum in the x-y plane for $\Omega = \omega_p/6$. First mark the positions of the pendulum at the times t_n (note the signs of x and y!). Then connect the entered positions in the correct order.

Explain qualitatively how the movement of the pendulum changes if it is in Stuttgart or Sydney instead of the North Pole.

Problem 8.3: Centrifugal governor

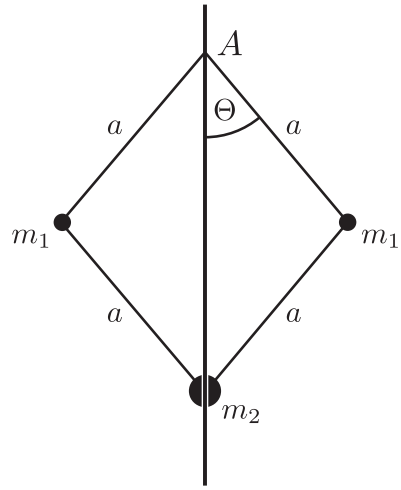
[**Oral** | 3 pt(s)]

ID: ex_centrifugal_governor:km25

Learning objective

In this exercise we practice the Lagrange formalism with generalized coordinates on the specific example of the centrifugal governor. This is another example where one can see that constraints are easily implemented in the Lagrange formalism.

The arrangement shown in the image turns with constant angular velocity ω around the vertical z-axis in a homogeneous gravitational field with gravitational acceleration g. The two masses m_1 are connected to the fixed point A and to the mass m_2 via massless rods of length a. The angle Θ is variable as well as the angle where two rods connect at the masses m_1. The mass m_2 can move freely along the z-axis.



a) Choose Θ as a generalized coordinate and find the Lagrangian. Is the energy H conserved? **1** pt(s)

b) Find the effective potential of the system. **1** pt(s)

c) Investigate the stability of the stationary solutions ($\Theta = \text{const.}$). Above which critical angular velocity ω_c will the masses m_1 move outwards from the z-axis? At what angle Θ is their stable state? **1** pt(s)