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Problem 7.1: Inclined plane

[**Written** | 4 pt(s)]

ID: ex_inclined_plane:km25

Learning objective

Constraints can be built directly into the Lagrange formalism. A simple and yet very instructive example is a particle on an inclined plane. The aim of this exercise is to understand the relationship between constraints, generalized coordinates and constraining forces.

A particle with mass m slides in the gravitational potential $V = mgy$ on an inclined plane with angle α without friction.

a) Specify the constraint $g(x, y) = 0$ and choose a suitable generalized coordinate. Check that the constraint is trivially satisfied in these coordinates and set up the Lagrange function. Then derive the equation of motion and solve it. **1** pt(s)

b) Calculate the resulting constraining force $\boldsymbol{Z} = m\ddot{\boldsymbol{r}} + \nabla V$ of the system. Make sure that the constraining force is perpendicular to the plane and corresponds to the expected force of the plane on the sphere. **1** pt(s)

c) The inclined plane is now replaced by a parabola $y = cx^2$. We only consider the case of small deflections x and a small curvature c. Proceed as in part (a) to determine the motion of the particle. **1** pt(s)

Hints: First set up the exact Lagrange function and then neglect the smallest term (for small x and small c).

d) Now consider a cylinder with mass m, radius R and moment of inertia I. This now rolls frictionless and without slippage on the inclined plane with angle α in the gravitational potential. Write down all constraints and choose generalized coordinates for which they are trivially satisfied. Then set up the Lagrange function and solve the Euler-Lagrange equation. **1** pt(s)

Finally, consider the special case of a hollow and a solid cylinder with moments of inertia $I_{\text{hollow}} = mR^2$ and $I_{\text{solid}} = mR^2/2$.

Problem 7.2: Sliding pendulum

[**Oral** | 4 pt(s)]

ID: ex_sliding_pendulum:km25

Learning objective

In this exercise we practice how the choice of suitable generalized coordinates can simplify problems with constraints in the Lagrange formalism.

We consider a pendulum with mass m_2 and coordinates $\boldsymbol{r}_2 = (x_2, y_2)^T$, whose suspension point can move along a horizontal straight line. The suspension point has the mass m_1 and coordinates $\boldsymbol{r}_1 = (x_1, y_1)^T$. The length of the pendulum rod is L and its mass can be neglected.

a) Formulate the constraints of this system and choose suitable generalized coordinates. Then set up the Lagrange function in the generalized coordinates. **1**pt(s)

b) Now set up the Euler-Lagrange equations and identify the resulting conservation variable. **1**pt(s)

c) To solve this differential equation, we restrict ourselves to the special case of small deflections of the pendulum $\phi \ll 1$, for which we can use the small-angle approximation for the cosine and sine. **1**pt(s)

   Hint: Neglect all terms of order ϕ^2 or higher.

d) Now we want to solve the exact Euler-Lagrange equations from part (b) numerically. **1**pt(s)

   To do this, we must first bring the system of differential equations to the form $\dot{\boldsymbol{\xi}} = \boldsymbol{f}(\boldsymbol{\xi})$. Show that the following system of differential equations follows from the Euler-Lagrange equations from (b):

$$\dot{x} = v\,, \tag{1}$$

$$\dot{v} = \frac{m_2 \sin(\phi)}{m_1 + m_2 \sin^2(\phi)} \left[g\cos(\phi) + L\omega^2 \right]\,, \tag{2}$$

$$\dot{\phi} = \omega\,, \tag{3}$$

$$\dot{\omega} = \frac{-\sin(\phi)}{m_1 + m_2 \sin^2(\phi)} \left[\frac{g}{L}(m_1 + m_2) + m_2 \cos(\phi)\omega^2 \right]\,. \tag{4}$$

   Hint: The coordinates of the suspension point and the pendulum are given by: $\boldsymbol{r}_1 = (x, 0)^T$ and $\boldsymbol{r}_2 = (x + L\sin\phi, -L\cos\phi)^T$.

   Now implement these equations of motion in the programming language of your choice (Mathematica, Matlab, Python, ...) and represent the motion of the sliding pendulum for $m_1 = 2$, $m_2 = 1$ and $L = 1$ as well as the following initial conditions:

   (i) The suspension point and the pendulum start at rest, but with a deflection angle $\phi(t = 0) = 60\,°$.

   (ii) The suspension point and the pendulum start with the same x-coordinate $x_1(t = 0) = x_2(t = 0) = 0$, but the suspension point has a starting speed $\dot{x}_1(t = 0) = 1$ while the pendulum is at rest at the beginning ($\dot{x}_2(t = 0) = 0$).

   Hint: Choose $\boldsymbol{\xi} = (x, v, \phi, \omega)^T$ and use the Euler method to numerically solve the differential equation $\dot{\boldsymbol{\xi}} = \boldsymbol{f}(\boldsymbol{\xi})$. I.e. start with the initial conditions $\boldsymbol{\xi}_0$ and calculate the new coordinates after a time step Δt by $\boldsymbol{\xi}_{k+1} = \boldsymbol{\xi}_k + \Delta t \boldsymbol{f}(\boldsymbol{\xi}_k)$. (Choose $\Delta t = 10^{-3}$, and show the movements for a total time $T = 3$).

Problem 7.3: Noether's theorem with AI

[**Oral** | 1 pt(s)]

ID: ex_noether_with_ai:km25

Learning objective

Certainly! Here's an exercise designed for undergraduate physics students. The goal of the exercise is to understand the possibilities and limitations of artificial intelligence in solving problems in theoretical physics and to deepen your understanding of Noether's theorem.

We consider a transformation that changes the Lagrangian function by only one total derivative

$$L(q^i, \dot{q}^i, t) = L'(h^i, \dot{h}^i, t) = L(h^i, \dot{h}^i, t) + \frac{d}{dt}F(h^i, t),$$

as in Problem 4.1. Here, q^i are the old coordinates and h^i are the new coordinates. Use an AI-tool of your choice to proof Noether's theorem and carefully check the result. Correct the AI if necessary. If the proof is correct try to convince the AI that it is wrong. Document your conversation with the AI.