

Problem 14.1: One-dimensional movement*

[Written | 10 bonuspt(s)]

ID: ex_eindimensionale_bewegung_3:km25

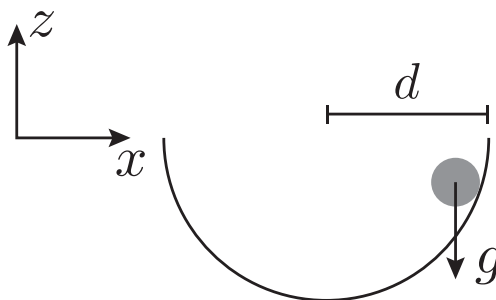
A point of mass with mass $m = 1$ moves along the coordinate x (1D) in a potential of the form $V(x) = 2x^2 - x^4 - 1$.

- Determine the equilibrium points and analyze their stability properties. 2pt(s)
- Sketch the phase space curves for energy $E = 0$ and one curve each for $E > 0$ and $E < 0$. 2pt(s)
- Write down the Lagrange function of the system. What is the Euler-Lagrange equation? Derive the equation of motion of the system. 2pt(s)
- Develop the equation of motion in lowest order around the stable equilibrium point and find the solution of the equation of motion with initial condition $x = 0.1$ and $\dot{x} = 0$. 2pt(s)
- Find the exact solution of the particle motion $x(t)$ at energy $E = 0$. 2pt(s)

Problem 14.2: Constraining forces*

[Written | 14 bonuspt(s)]

ID: ex_zwangskraefte:km25



A cylinder with mass M and radius R moves frictionless on a curved surface under the influence of gravity. The surface has the shape of a semicircle (see sketch above) and is determined by the equation $z = -\sqrt{d^2 - x^2}$.

- First consider the case with a frictionless sliding cylinder. Set up the constraints and introduce a suitable generalized coordinate. 2pt(s)
- Derive the Lagrange function for the generalized coordinate. What does the equation of motion look like? 2pt(s)

Next, consider the case of a rolling cylinder that rolls on the surface without slip and without rolling friction.

- Derive the general formula for the moment of inertia of the cylinder as a function of its mass M (along its axis of symmetry). The cylinder has the homogeneous density ρ , the radius R and the length l . 2pt(s)

- d) Write down the kinetic energy T for the rolling cylinder and determine the potential energy V . 2pt(s)
- e) Set up the constraint for the cylinder to roll. Introduce a suitable generalized coordinate to describe the motion of the cylinder. 2pt(s)
- f) Determine the Lagrange function in the generalized coordinate. 2pt(s)
- g) Compare the dynamics of the cylinder in the sliding and rolling cases. How do the equations of motion change? What are the oscillation frequencies around the stable equilibrium point at small deflections? 2pt(s)

Problem 14.3: Hamilton formalism*

[Written | 10 bonuspt(s)]

ID: ex_hamilton_formalismus:km25

The dynamics of a system with the generalized coordinates $q_1(t)$ and $q_2(t)$ is described by the Lagrange function

$$L(q_1, q_2, \dot{q}_1, \dot{q}_2) = \frac{m}{1 + (q_1 + q_2)^2} [\dot{q}_1^2 + \dot{q}_2^2] - V_0 (q_1 + q_2)^4. \quad (1)$$

- a) Determine the canonical momentum p_1, p_2 and derive the Hamilton function. 3pt(s)
- b) Write down the equations of motion in Hamiltonian formalism. 3pt(s)
- c) Determine the Poisson's bracket $\{F, H\}$ of $F(t) = p_1 - p_2$. Describe the role of the Poisson's bracket. 4pt(s)

Problem 14.4: Theory of relativity*

[Written | 7 bonuspt(s)]

ID: ex_relativitaetstheorie:km25

- a) Describe the phenomenon of time dilation. Justify the formula for time dilation using a Lorentz transformation. 3pt(s)
- b) Consider a point of mass with mass m and velocity v . Write down the individual components of its relativistic momentum p^μ and determine $p_\mu p^\mu$. 2pt(s)
- c) The point of mass is initially at rest and undergoes an energy transfer of $\Delta E = 2mc^2$ due to an elastic collision. Determine the velocity and the relativistic momentum of the point of mass after the collision. 2pt(s)