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Information on lecture and tutorials

Here a few infos on the modalities of the course ***"Theo I: Klassische Mechanik"***:

- The `C@MPUS-ID` of this course is 042010002.
- You can find detailed information on lecture and tutorials on the website of our institute:
  `https://itp3.info/km25`
- You can also find detailed information on lecture and tutorials on `ILIAS`:
  `https://ilias3.uni-stuttgart.de/go/crs/4029824`
- **Written** problems have to be handed in via `ILIAS` or during the tutorial and will be corrected by the tutors. You must earn at least **80 %** of the written points to be admitted to the exam.
- **Oral** problems have to be prepared for the exercise session and will be presented by a student at the blackboard. You must earn at least **66 %** of the oral points to be admitted to the exam.
- Every student is required to **present** at least **2** of the oral problems at the blackboard to be admitted to the exam.
- Problems marked with an asterisk ($*$) are optional and can earn you bonus points.
- If you have questions regarding the problem sets, feel free to contact your tutor at any time.

Problem 1.1: Galilean Transformations I

[**Written** | 2 pt(s)]

ID: ex_galilei_transformation_I:km25

Learning objective

Galilean transformations were introduced in the lecture. In this task, the transformation behavior of momentum and kinetic energy is to be investigated. In addition, the shape invariance of Newton's equations is to be checked.

The Galilean transformations describe the transition between inertial frames of reference. They can be written as

$$t' = t + a, \tag{1a}$$

$$\boldsymbol{r}' = R\boldsymbol{r} + \boldsymbol{u}t + \boldsymbol{b}. \tag{1b}$$

Here $R \in \mathrm{SO}(3)$ is a 3×3- matrix with determinant 1 and $R^T R = \mathbb{1}$, $\boldsymbol{u}, \boldsymbol{b} \in \mathbb{R}^3$ vectors and $a \in \mathbb{R}$ a scalar. In addition, there are the discrete Galilean transformations: the spatial reflection $\boldsymbol{r}' = -\boldsymbol{r}$ and the time reversal $t' = -t$.

a) Calculate how the momentum $\mathbf{p} = m\mathbf{v}$ and the kinetic energy $T = m\mathbf{v}^2/2$ transform under a Galilean transformation. **1**pt(s)

Consider two point particles interacting with the potential $V(|\mathbf{r}_1 - \mathbf{r}_2|)$. The motion of the particles is described by Newton's equations

$$m_1 \ddot{\boldsymbol{r}}_1 = \boldsymbol{F}_{12}(\mathbf{r}_1, \mathbf{r}_2), \tag{2a}$$
$$m_2 \ddot{\boldsymbol{r}}_2 = \boldsymbol{F}_{21}(\mathbf{r}_1, \mathbf{r}_2) \tag{2b}$$

where $\boldsymbol{F}_{12}(\mathbf{r}_1, \mathbf{r}_2)$ and $\boldsymbol{F}_{21}(\mathbf{r}_1, \mathbf{r}_2)$ are the forces resulting from $V(|\mathbf{r}_1 - \mathbf{r}_2|)$.

b) Show that Newton's equations for the system are form-invariant under Galilean transformation (including the discrete transformations of reflection and time reversal). **1**pt(s)

Hint: First consider how acceleration and force transform under Galilean transformation. What do the resulting equations of motion look like?

Problem 1.2: Galilean Transformations II

[**Written** | 5 pt(s)]

ID: ex_galilei_transformation_II:km25

Learning objective

The aim of this task is to prove that the Galilean transformations form a non-commutative group.

A set $\mathcal{G}$ together with a map (usually called multiplication) $\cdot : \mathcal{G} \times \mathcal{G} \to \mathcal{G}$ is a group if the following conditions are met.

- For any three elements $g, g', g'' \in \mathcal{G}$ it holds that $g \cdot (g' \cdot g'') = (g \cdot g') \cdot g''$. (associativity)
- There exists a neutral element $e \in \mathcal{G}$ with $g \cdot e = e \cdot g = g$.
- For every element $g \in \mathcal{G}$ there exists an inverse element $g^{-1} \in \mathcal{G}$, so that $g^{-1} \cdot g = g \cdot g^{-1} = e$.

The discrete transformations can be neglected for the entire task.

a) First find the group multiplication. To do this, carry out a Galilean transformation g. Then apply another Galilean transformation g' to the transformed coordinates. How do the new coordinates depend on the original coordinates? Compare your result with a Galilean transformation g''. Determine $g'' = (R'', \boldsymbol{u}'', \boldsymbol{b}'', a'')$ as a function of $g = (R, \boldsymbol{u}, \boldsymbol{b}, a)$ and $g' = (R', \boldsymbol{u}', \boldsymbol{b}', a')$. **1**pt(s)

Now show that the Galilean transformations together with the previously determined connection form a group.

b) Show that the associativity is fulfilled. **1**pt(s)

c) Find the neutral element $e \in \mathcal{G}$ so that $g \cdot e = e \cdot g = g$ is satisfied. **1**pt(s)

d) For each element $g \in \mathcal{G}$ determine an inverse element $g^{-1} \in \mathcal{G}$, so that $g^{-1} \cdot g = g \cdot g^{-1} = e$. **1**pt(s)

A group is called a commutative group if it also holds for all elements $g, g' \in \mathcal{G}$ that $g \cdot g' = g' \cdot g$. (commutativity)

e) Show that the Galilean transformations do not form a commutative group. **1**pt(s)

Hint: Find an example of two Galilean transformations that violates commutativity.

Problem 1.3: Galilei Invariance

[**Oral** | 3 pt(s)]

ID: ex_galilei_invariance:km25

Learning objective

In this exercise, we use the invariance of Newton's equations under Galilean transformation to make some general statements about the motion of point particles.

Consider a mechanical system with an arbitrary force law. The force acting on the ith particle can generally be written as $\boldsymbol{F}_i(\boldsymbol{r}_1, \boldsymbol{r}_2, ..., \dot{\boldsymbol{r}}_1, \dot{\boldsymbol{r}}_2, ..., t)$.

a) The system initially consists of two point particles. At the initial time, they are at rest (in a certain inertial frame). Show that the motion follows the straight line containing the initial positions. **1** pt(s)

Hints:

- Note that Newton's equations are invariant under Galilean transformation. What conditions follow from this for the force? Consider the invariance under temporal translation, spatial translation and rotation.
- What symmetries does the system have? (Under which transformations is it invariant?)

b) Show that for three point particles initially at rest, the movement is restricted to a plane. **1** pt(s)

c) Consider a mechanical system of two point particles with arbitrary initial velocities. Show that, in a suitable inertial system, the motion takes place in a plane. **1** pt(s)