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Problem 8.1: Four level system degenerate perturbation theory

[**Written** | 4 pt(s)]

ID: ex_degenerate_pt_4level_system:fqt2425

> **Learning objective**
>
> Within a four level-system scenario, we will use degenerate perturbation theory to obtain the corrections to the eigenenergies of the Hamiltonian under the influence of a perturbation. As we will see, in some cases first-oder perturbation theory is already enough to capture the exact eigenvalues of a physical system.

Consider a generic four level system with an unperturbed Hamiltonian $\hat{H}_0$ given by

$$\hat{H}_0 = E_0 \begin{pmatrix} 15 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix} \tag{1}$$

which is subject to a perturbation $\hat{H}_1$ given by

$$\hat{H}_1 = \frac{E_0}{100} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} . \tag{2}$$

a) Find the eigenstates of the unperturbed Hamiltonian $\hat{H}_0$ as well as the exact eigenvalues of the total Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_1$. **2**pt(s)

b) Find the eigenenergies of $\hat{H}$ to first and second order perturbation order. How do they compare with the exact values obtained in (a)? **2**pt(s)

Problem 8.2: Perturbing the one-dimensional harmonic oscillator

[**Oral** | 2 pt(s)]

ID: ex_time_dep_pt_1d_harmonic_osc:fqt2425

> **Learning objective**
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> This exercise exemplifies time-dependent perturbation theory for the canonical example of a one-dimensional harmonic oscillator under the influence of a constant perturbing potential during a certain time interval.

A particle of mass m is in the ground state of a one-dimensional harmonic oscillator potential. The oscillator frequency is ω. At $t = 0$, a weak constant force $\mathcal{F}$ is applied and acts until time $t = \tau$ such that the perturbing potential is given by

$$\hat{W}(x, t) = -\mathcal{F}x, 0 < t < \tau. \tag{3}$$

Use first-order time-dependent perturbation theory to find the value (or values) of τ, call them τ_{max}, that maximize the probability of a transition to the first excited state ($n = 1$).

Problem 8.3: Perturbing the hydrogen atom with a time-dependent electric field [Oral | 4 pt(s)]

ID: ex_time_dep_pt_hydrogen_electric_field:fqt2425

Learning objective

This exercise intends to illustrate how a time-dependent perturbation can induce transitions between states. More importantly, we will see in the case of spherically symmetric problems that the probability of certain transitions can be determined without explicit calculations by simply exploiting symmetry arguments.

A hydrogen atom in the ground state is immersed in an electric field F that is constant in the z-direction, but varies in time as

$$F = \begin{cases} 0, & \text{for } t < 0. \\ F_0 e^{-t/\tau}, & \text{for } t > 0. \end{cases} \tag{4}$$

a) Use time-dependent perturbation theory to determine the probability that after a long time, i.e., $t \to \infty$, the atom will be in the $n = 2, \ell = 1, m = 0$ state. **2**pt(s)

b) Suppose now that the final state had been chosen to be $|200\rangle$ instead of $|210\rangle$. What is the probability that after a long time the electric field perturbation will induce a transition to this state? **2**pt(s)

   Hint: Exploit the parity of the spherical harmonics $Y_{\ell,m}(\theta, \phi)$.