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Problem 6.1: Dielectric half space

[**Oral** | 2 pt(s)]

ID: ex_dielectric_half_space:

Learning objective

We have previously learned about the method if image charges. In this exercise we will apply the knowledge of the image charges in order to calculate the electric field inside the dielectric media.

Consider two linear dielectric half spaces, which have the y-z-plane as common interface. For $x > 0$ there is the dielectric constant ε_1 and for $x < 0$ there is ε_2. Within the media 1 (ε_1) there is a point charge q. Calculate the **E** - and **D** -fields for the two cases $\varepsilon_1 > \varepsilon_2$ and $\varepsilon_1 < \varepsilon_2$ by using the methods of image charges. In addition, sketch the **E** - and **D** -fields for both cases.

Hints: Use the following Ansatz for the potentials at a point P described by cylindrical coordinates (ρ, φ, x):

$$\Phi_1(\rho, x) = \frac{1}{4\pi\varepsilon_1}\left(\frac{q}{R_1} + \frac{q'}{R_2}\right), \quad x > 0\,, \tag{1}$$

and

$$\Phi_2(\rho, x) = \frac{1}{4\pi\varepsilon_2}\frac{q''}{R_1}, \quad x < 0\,, \tag{2}$$

where $R_1 = \sqrt{\rho^2 + (d-x)^2}$ and $R_2 = \sqrt{\rho^2 + (d+x)^2}$, assuming that the charge q and its image charge are positioned symmetrically around the $x = 0$ plane.

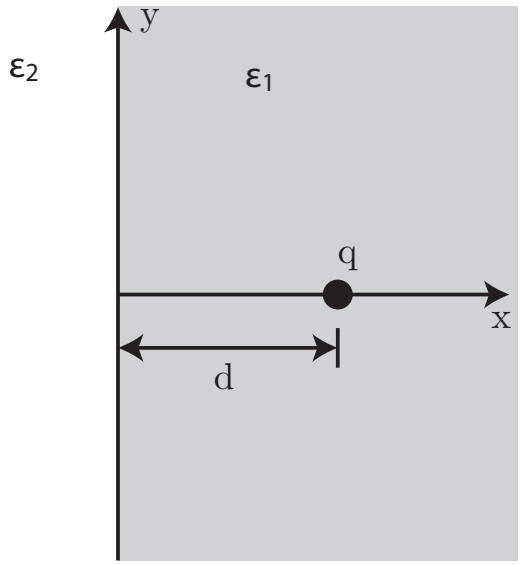

Figure 1: Half spaces separated by an interface, including a charge q.

Problem 6.2: Parallel plate capacitor with a dielectric

[**Oral** | 3 pt(s)]

ID: ex_parallel_plate_capacitor_with_a_dielectric:

Learning objective

Within this exercise we analyse the influence of the dielectric media on the plate capacitor.

Consider a parallel plate capacitor with quadratic plates of edge length l and distance d between the plates. The capacitor is directly connected to a battery with voltage V.

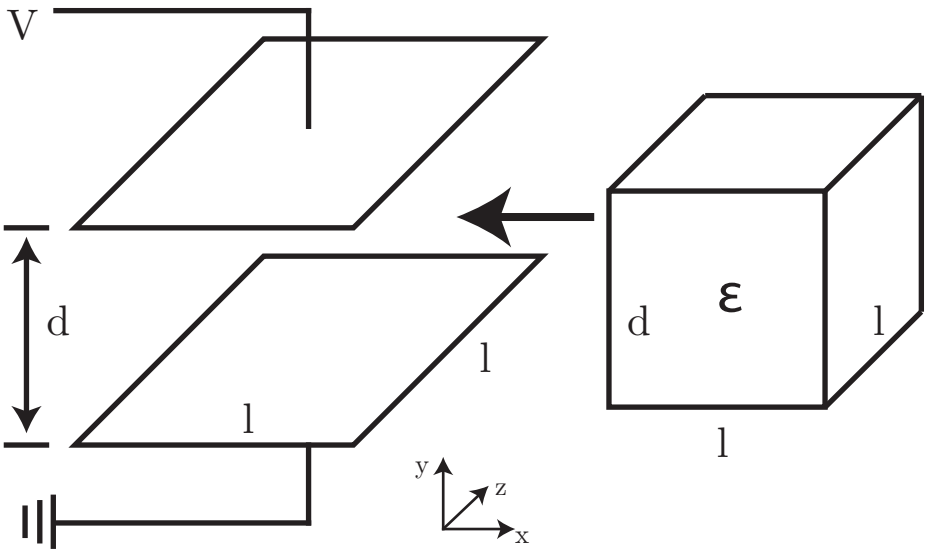

Figure 2: Parallel plate capacitor with dielectric media.

a) What is the total charge Q, respectively $-Q$ of both plates? 1 pt(s)

b) Now the dielectric media, length of the edges $\{d \times l \times l\}$ and isotropic dielectric constant $\varepsilon > 1$, is inserted into the capacitor, right in between the parallel plates. How does Q change, if the battery is still connected? What force acts on the dielectric media and in which direction? 1 pt(s)

c) This time we disconnect the plates from the battery before we insert the dielectric media. What happens now to the potential difference V and what force acts now? 1 pt(s)

Hint: Neglect boundary effects, i.e. consider a homogeneous electric field between the plates, which vanishes outside the plates.

Problem 6.3: Point charge near a dielectric sphere

[**Written** | 3 pt(s)]

ID: ex_dieletric_sphere:

Learning objective

In this exercise, we will find the electrostatic potential of a point charge in the presence of a dielectric

sphere using an expansion in spherical harmonics, and verify that the conducting sphere result is recovered in the appropriate limit.

A point charge q is located in free space a distance d from the center of a dielectric sphere of radius a ($a < d$) and dielectric constant ϵ/ϵ_0.

Hint: Place the sphere at the origin and the point charge on the z-axis at $z = d$. The problem has azimuthal symmetry, so the solution involves only ordinary Legendre polynomials. Inspired by the conducting sphere problem, try an image charge q' at $z = a^2/d$ for the exterior region and a single image charge q'' at $z = d$ for the interior region.

a) Find the potential at all points in space as an expansion in spherical harmonics. **1**pt(s)

b) Calculate the rectangular components of the electric field near the center of the sphere. **1**pt(s)

c) Verify that, in the limit $\epsilon/\epsilon_0 \to \infty$, the result reduces to that of a conducting sphere. **1**pt(s)