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Problem 4.1: Cylindrical Capacitor [**Oral** | 1 pt(s)]

ID: ex_cylindrical_capacitor:edyn26

> **Learning objective**
>
> In this exercise, we apply our knowledge to evaluate the capacity of a cylindrical capacitor. Given the translational invariance of the cylinder, we can apply the tools from the previous lecture to find the electric field and charge distribution.

Consider two concentric conducting cylinders of length L with radii $R_1 < R_2$ (coaxial cable) that are separated from each other by an insulating medium such as air or vacuum. Compute the capacitance C per unit length for this setup for the case of very long conductors $L \gg R_2$. How does the capacitance change if the inner cylinder is not hollow, but solid instead ?

Problem 4.2: Legendre polynomials and charge density [**Oral** | 3 pt(s)]

ID: ex_legendre_polynomials_and_charge_density:edyn26

> **Learning objective**
>
> Boundary value problems with a surface given by a sphere can naturally be solved in spherical coordinates. Here, we apply this method to solve the electrostatic problem with Dirichlet boundary conditions on a sphere, and get acquainted with Legendre Polynomials.

Let a spherical shell with radius R carry a fixed charge density $\sigma(\theta)$ such that the potential on the sphere is

$$V(r = R, \theta) = V_0 + V_1 \cos\theta + V_2 \cos 2\theta \tag{1}$$

where V_0, V_1, and V_2 are constants and θ is the polar angle.

a) Find the potential $V(r, \theta)$ inside and outside of the spherical shell ($V(\infty) = 0$). Use the ansatz **1**pt(s)

$$\Phi(r, \theta) = \sum_{l=0}^{\infty} (a_l r^l + b_l r^{-(l+1)}) P_l(\cos\theta), \tag{2}$$

where $P_l(x)$ are the Legendre polynomials.

b) From the potential $V(r, \theta)$ compute the electric field $\boldsymbol{E}(r, \theta)$. **1**pt(s)

c) Oberserve how - as is to be expected - the component of $\boldsymbol{E}$ perpendicular to the spherical shell is discontinuous at $r = R$ with a jump of $\boldsymbol{E}_\perp^{\text{outside}} - \boldsymbol{E}_\perp^{\text{inside}} = 4\pi K\sigma(\theta)$ while the tangential component is continuous. Use this to compute the surface charge distribution $\sigma(\theta)$. **1**pt(s)

Hint: The first Legendre polynomials are given by

$$P_0(x) = 1, \;\; P_1(x) = x, \;\; P_2(x) = \frac{1}{2}(3x^2 - 1) \tag{3}$$

Furthermore they obey the orthogonality relation

$$\int_{-1}^{1} dx P_l(x) P_m(x) = \frac{2}{2l+1}\delta_{lm}. \tag{4}$$

Problem 4.3: Potential of an infinitely long cylinder

[**Written** | 2 pt(s)]

ID: ex_potential_infinite_cylinder:edyn26

> **Learning objective**
>
> Here, we solve Laplace's equation in cylindrical coordinates with Dirichlet boundary conditions.

We consider an infinitely long, hollow cylinder of radius R. Using Laplace's equation in cylindrical coordinates, we determine the electric potential inside and outside of the cylinder, given the value of the potential on the boundary of the cylinder.

a) Assume the potential on the boundary is given by **1**pt(s)

$$\phi(z, \varrho = R, \varphi) = \phi_0 + \phi_1 \cos\varphi \;, \tag{5}$$

where z is the axial coordinate, φ is the polar angle, and ϱ the radial distance in cylindrical coordinates. Think about the geometry of the problem and calculate the potential inside and outside of the cylinder.

b) The potential on the boundary of the cylinder is **1**pt(s)

$$\phi(z, \varrho = R, \varphi) = \cos(kz)\left(\phi_0 + \phi_1 \cos\varphi\right) \;, \tag{6}$$

with $k \neq 0$. Calculate the potential and determine its value in the limit $\varrho \to \infty$ (for taking this limit, it is helpful to look up the asymptotic behavior of the Bessel function e.g. on Wikipedia).