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Problem 6.1: Magnetic moment of a rotating spherical shell [**Written** | 3 pt(s)]

ID: ex_magnetic_moment_of_a_rotating_spherical_shell:edyn23

> **Learning objective**
>
> Within this exercise we will analyse the properties of the rotating charged spherical shell. We will calculate the magnetic field created by the spherical shell, and calculate the interaction energy between two identical spherical shells. We will see that the force exerted on the second sphere by the magnetic field of the first sphere depends on their angular velocities.

A spherical shell of radius R and charge Q (homogeneously distributed on the surface) is rotating around its z-axis with angular velocity $\boldsymbol{\omega} = \omega \boldsymbol{e}_z$.

a) Calculate the current density $\boldsymbol{j}(\boldsymbol{r}) = \boldsymbol{v}(\boldsymbol{r})\rho(\boldsymbol{r})$ and calculate the magnetic moment $\boldsymbol{m} = \frac{1}{2}\int d^3r(\boldsymbol{r} \times \boldsymbol{j}(\boldsymbol{r}))$ of the spherical shell. **1**pt(s)

b) Show explicitly that the magnetic field created by the rotating charged spherical shell at point $\boldsymbol{r} \gg R$, can be expressed as: **1**pt(s)

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\left(\frac{3(\boldsymbol{r}\cdot\boldsymbol{m})\boldsymbol{r}}{|\boldsymbol{r}|^5} - \frac{\boldsymbol{m}}{|\boldsymbol{r}|^3}\right) = \frac{\mu_0 Q R^2}{12\pi}\left(\frac{3(\boldsymbol{r}\cdot\boldsymbol{\omega})\boldsymbol{r}}{|\boldsymbol{r}|^5} - \frac{\boldsymbol{\omega}}{|\boldsymbol{r}|^3}\right). \quad (1)$$

c) Now let $\boldsymbol{r}_2$ be a vector such that $\boldsymbol{r}_2 \perp \boldsymbol{\omega}$ and $|\boldsymbol{r}_2| \gg R$. Assume that we have an identical sphere at $\boldsymbol{r}_2$ rotating at angular velocity $\boldsymbol{\omega}_2$ parallel to $\boldsymbol{\omega}$. Calculate the interaction energy between the magnetic moment $\boldsymbol{m}_1$ of the first spherical shell and the magnetic moment $\boldsymbol{m}_2$ of the second spherical shell. Calculate the force exerted by the magnetic field on the spherical shell at $\boldsymbol{r}_2$. **1**pt(s)

   Hint: Due to the large distance between the spheres you can approximate them as two point-like objects carrying some magnetic moment.

Problem 6.2: Parallel plate capacitor with a dielectric [**Oral** | 3 pt(s)]

ID: ex_parallel_plate_capacitor_with_a_dielectric:edyn23

> **Learning objective**
>
> Within this exercise we analyse the influence of the dielectric media on the plate capacitor.

Consider a parallel plate capacitor with quadratic plates of edge length l and distance d between the plates. The capacitor is directly connected to a battery with voltage V.

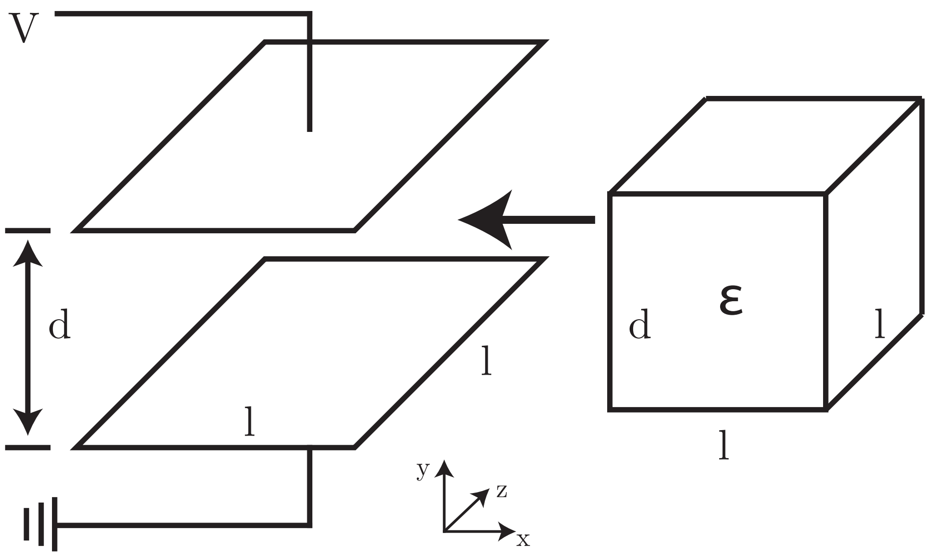

Abbildung 1: Parallel plate capacitor with dielectric media.

a) What is the total charge Q, respectively $-Q$ of both plates? **1**pt(s)

b) Now the dielectric media, length of the edges $\{d \times l \times l\}$ and isotropic dielectric constant $\varepsilon > 1$, is inserted into the capacitor, right in between the parallel plates. How does Q change, if the battery is still connected? What force acts on the dielectric media and in which direction? **1**pt(s)

c) This time we disconnect the plates from the battery before we insert the dielectric media. What happens now to the potential difference V and what force acts now? **1**pt(s)

Hint: Neglect boundary effects, i.e. consider a homogeneous electric field between the plates, which vanishes outside the plates.

Problem 6.3: Dielectric half space [**Oral** | 2 pt(s)]

ID: ex_dielectric_half_space:edyn23

Learning objective

We have previously learned about the method if image charges. In this exercise we will apply the knowledge of the image charges in order to calculate the electric field inside the dielectric media.

Consider two linear dielectric half spaces, which have the y-z-plane as common interface. For $x > 0$ there is the dielectric constant ε_1 and for $x < 0$ there is ε_2. Within the media 1 (ε_1) there is a point charge q. Calculate the **E** - and **D** -fields for the two cases $\varepsilon_1 > \varepsilon_2$ and $\varepsilon_1 < \varepsilon_2$ by using the methods of image charges. In addition, sketch the **E** - and **D** -fields for both cases.

Hints: Use the following Ansatz for the potentials at a point P described by cylindrical coordinates (ρ, φ, x):

$$\Phi_1(\rho, x) = \frac{1}{4\pi\varepsilon_1}\left(\frac{q}{R_1} + \frac{q'}{R_2}\right), \quad x > 0\,, \tag{2}$$

and

$$\Phi_2(\rho, x) = \frac{1}{4\pi\varepsilon_2}\frac{q''}{R_1}\,, \quad x < 0\,, \tag{3}$$

where $R_1 = \sqrt{\rho^2 + (d-x)^2}$ and $R_2 = \sqrt{\rho^2 + (d+x)^2}$, assuming that the charge q and its image charge are positioned symmetrically around the $x = 0$ plane.

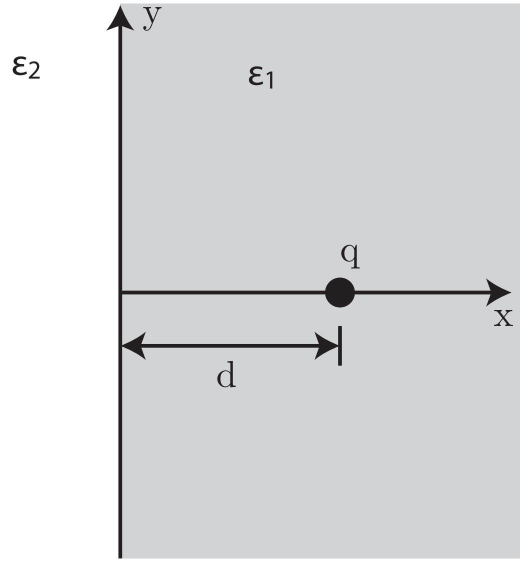

Abbildung 2: Half spaces separated by an interface, including a charge q.