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November 26th, 2025
WS 2025/26

Problem 7.1: Perturbing the harmonic oscillator with a weak electric field [**Oral** | 6 pt(s)]

ID: ex_charged_oscillator_in_electric_field_stationary_perturbation:aqt2526

Learning objective

We revisit the problem of a charged oscillating particle under the influence of an electric field. Although this problem was solved exactly previously, it is instructive to see that stationary perturbation theory produces consistent results.

A particle of charge q and mass m, which is moving in a one-dimensional harmonic potential of frequency ω, is subject to a *weak* electric field $\mathcal{E}$ in the x-direction. As we have seen, the Hamiltonian governing the interaction between the oscillating charge and the external electric field can be written as

$$\hat{H} = \hat{H}_0 + \hat{H}_p, \quad \hat{H}_0 = -\frac{\hbar^2}{2m}\frac{d^2}{dX^2} + \frac{1}{2}m\omega^2 X^2, \quad \hat{H}_p = q\mathcal{E}X. \tag{1}$$

a) First, consider the general theory of non-degenerate stationary perturbation theory. **2**pt(s)

(i) Explain and write down the equations on how you would calculate corrections for the eigenvalues and eigenfunctions up to second and first order, respectively.

(ii) What is the condition for λ that ensures the considered corrections in the perturbative expansion are small?

$$\begin{aligned} E_n &= E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \dots, \\ |\psi_n\rangle &= \left|n^{(0)}\right\rangle + \lambda \left|n^{(1)}\right\rangle + \dots, \end{aligned} \tag{2}$$

where $\hat{H}_0 \left|n^{(0)}\right\rangle = E_n^{(0)} \left|n^{(0)}\right\rangle$ and $\hat{H}_p = \lambda \hat{W}$.

As a consistency check, what happens to this convergence condition if there are unperturbed energy levels E_n^0 which are degenerate?

b) Use the previously explained formalism to calculate the first nonzero energy correction to (1), and compare it with the exact result obtained in exercise 2.2 in problem sheet 2. **2**pt(s)

c) Calculate the correspondent corrections to the eigenfunctions up to first order in perturbation theory. **2**pt(s)

Problem 7.2: The Stark Effect [**Oral** | 4 pt(s)]

ID: ex_stark_effect_nondegenerate_pt:aqt2526

Learning objective

In this task, we will see the effect of an electric field on the ground state of the hydrogen atom and explore its polarizability using nondegenerate perturbation theory.

The effect that an external electric field has on the energy levels of an atom is called the Stark effect. In the absence of an electric field, the (unperturbed) Hamiltonian of the hydrogen atom (in CGS units) is

$$\hat{H}_0 = \frac{\hat{\boldsymbol{p}}^2}{2\mu} - \frac{e^2}{r}.$$

The eigenfunctions of this Hamiltonian, $\psi_{nlm}(\boldsymbol{r})$, are given by

$$\langle r\theta\varphi \mid nlm\rangle = \psi_{nlm}(r,\theta,\varphi) = R_{nl}(r)Y_{lm}(\theta,\varphi).$$

a) Consider an external uniform weak electric field, which is directed along the positive z-axis, $\boldsymbol{\mathcal{E}} = \mathcal{E}\boldsymbol{e}_z$, on the ground state of a hydrogen atom. Ignoring the spin degrees of freedom, calculate exactly the first order correction to the ground state energy, and write down the general expression for the second order correction. **2**pt(s)

b) The perturbed energy up to the first (generally) non-vanishing correction can be written as $E \approx E_0 - \boldsymbol{d}\cdot\boldsymbol{\mathcal{E}}$, where $\boldsymbol{d}$ is the electric dipole moment induced by the electric field $\boldsymbol{\mathcal{E}}$. Its components are related to $\boldsymbol{\mathcal{E}}$ by the polarizability tensor α_{ij}, $d_i = \alpha_{ij}\mathcal{E}_j + o(\mathcal{E})$. Hence, $E \approx E_0 - \alpha_{ij}\mathcal{E}_i\mathcal{E}_j + o(\mathcal{E}^2)$. Formally, the polarizability tensor can be defined as follows **2**pt(s)

$$\alpha_{ij} = -\left.\frac{\partial^2 E(\boldsymbol{\mathcal{E}})}{\partial\mathcal{E}_i\partial\mathcal{E}_j}\right|_{\boldsymbol{\mathcal{E}}=0}.$$

Since the ground state of the Hydrogen atom is isotropic, the polarizability tensor is diagonal $\alpha_{ij} = \alpha\delta_{ij}$. Find an upper bound for the polarizability α.
[**Hint**: Use $\langle 100|\hat{Z}|100\rangle = 0$ and that the set of states $|nlm\rangle$ is complete].

Problem 7.3: The Stark Effect revisited

[**Written** | 4 pt(s)]

ID: ex_stark_effect_degenerate_pt:aqt2526

Learning objective

We investigate the effects of a uniform, weak electric field on the excited states of the hydrogen atom using degenerate perturbation theory.

When the external electric field is turned on, some energy levels will split. The Hamiltonian of the electron (with $-e$ charge) subjected to the electric field directed along z-axis, $\boldsymbol{\mathcal{E}} = \mathcal{E}\boldsymbol{e}_z$, is given by

$$\hat{H}_p = e\mathcal{E}\hat{z}.$$

In the absence of any external electric field, the first excited state of the Hydrogen atom (i.e., $n = 2$) is fourfold degenerate: the states $|nlm\rangle = |200\rangle, |210\rangle, |211\rangle$, and $|21-1\rangle$ have the same energy $E_2 = -R_y/4$, where $R_y = \mu e^4/\left(2\hbar^2\right) = 13.6\text{eV}$ is the Rydberg constant.

a) In order to calculate the energy levels of the $n = 2$ states, first determine the matrix elements of the 4×4 Hamiltonian $\hat{H}_p$. **2**pt(s)

b) Diagonalize the matrix obtained above and determine the eigenenergies of $n = 2$ states up to (first order). Are all the degeneracies lifted? **2**pt(s)