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Problem 10.1: Particles in a Well

[**Oral** | 6 pt(s)]

ID: ex_in_distinguishabe_particles_in_1d_potential_well:

Learning objective

The spin-statistic theorem states that the many-body wave function for elementary particles with integer spin (bosons) is symmetric under the exchange of any two particles, whereas for particles with half-integer spin (fermions), the wave function is antisymmetric under such an exchange. In this exercise, we will use this theorem to determine the ground state and excited states of distinguishable and indistingishable particles in a 1D infinite potential well.

Consider a system of three non-interacting particles that are confined to move in a one-dimensional infinite potential well of length a such that $V(x) = 0$ for $0 < x < a$ and $V(x) = \infty$ for other values of x. Determine the energy and wave function of the ground state and the first and second excited states when the three particles are:

a) Spinless and distinguishable with masses $m_1 < m_2 < m_3$. **2**pt(s)

b) Identical scalar (spin 0) bosons. **2**pt(s)

c) Identical spin $\frac{1}{2}$ fermions. **2**pt(s)

Problem 10.2: Particles in a harmonic oscillator potential

[**Oral** | 4 pt(s)]

ID: ex_2_particles_system_harmonic_oscillator:

Learning objective

Similarly to the previous exercise, we will exploit the symmetrization of the wave functions for determining the ground state and excited states of a system made of two non-interacting identical particles under the influence of an external harmonic oscillator potential.

Consider a system of two non-interacting identical particles moving in a common external harmonic oscillator potential. Since the particles are non-interacting and identical, their Hamiltonian is given by $\hat{H} = \hat{H}_1 + \hat{H}_2$, where $\hat{H}_1$ and $\hat{H}_2$ are the Hamiltonians of particles 1 and 2:

$$\hat{H}_j = -\left(h^2/2m\right) d^2/dx_j^2 + m\omega x_j^2/2$$

with $j = 1, 2$. The total energy of the system is $E_{n_1 n_2} = \varepsilon_{n_1} + \varepsilon_{n_2}$, where $\varepsilon_{n_j} = \left(n_j + \frac{1}{2}\right) h\omega$.

a) We first consider two spin-1 particles. The spin states corresponding to $S = 2$ are given by **2**pt(s)

$$|2, \pm 2\rangle = |1, 1; \pm 1, \pm 1\rangle, \quad |2, \pm 1\rangle = \frac{1}{\sqrt{2}}(|1, 1; \pm 1, 0\rangle + |1, 1; 0, \pm 1\rangle),$$

$$|2, 0\rangle = \frac{1}{\sqrt{6}}(|1, 1; 1, -1\rangle + 2|1, 1; 0, 0\rangle + |1, 1; -1, 1\rangle),$$

those corresponding to $S = 1$ by

$$|1, \pm 1\rangle = \frac{1}{\sqrt{2}}(\pm|1,1;\pm 1,0\rangle \mp |1,1;0,\pm 1\rangle),$$
$$|1, 0\rangle = \frac{1}{\sqrt{2}}(|1,1;1,-1\rangle - |1,1;-1,1\rangle),$$

and the one corresponding to $S = 0$ by

$$|0, 0\rangle = \frac{1}{\sqrt{3}}(|1,1;1,-1\rangle - |1,1;0,0\rangle + |1,1;-1,1\rangle).$$

Determine the energy and the wave functions of the ground state and the first excited state for two spin 1 particles (bosons) with no orbital angular momentum.

b) Calculate the same quantities for two spin $\frac{1}{2}$ particles (fermions). Remember in this case that the singlet (anti-symmetric) and triplet (symmetric) states are given by: **2**pt(s)

$$\chi_{\text{triplet}}\ (\boldsymbol{S}_1, \boldsymbol{S}_2) = \begin{cases} \left|\frac{1}{2}\frac{1}{2}\right\rangle_1 \left|\frac{1}{2}\frac{1}{2}\right\rangle_2, \\ \frac{1}{\sqrt{2}}\left(\left|\frac{1}{2}\frac{1}{2}\right\rangle_1 \left|\frac{1}{2} - \frac{1}{2}\right\rangle_2 + \left|\frac{1}{2} - \frac{1}{2}\right\rangle_1 \left|\frac{1}{2}\frac{1}{2}\right\rangle_2\right), \\ \left|\frac{1}{2} - \frac{1}{2}\right\rangle_1 \left|\frac{1}{2} - \frac{1}{2}\right\rangle_2, \end{cases}$$

and:

$$\chi_{\text{singlet}}\ (\boldsymbol{S}_1, \boldsymbol{S}_2) = \frac{1}{\sqrt{2}}\left(\left|\frac{1}{2}\frac{1}{2}\right\rangle_1 \left|\frac{1}{2} - \frac{1}{2}\right\rangle_2 - \left|\frac{1}{2} - \frac{1}{2}\right\rangle_1 \left|\frac{1}{2}\frac{1}{2}\right\rangle_2\right).$$